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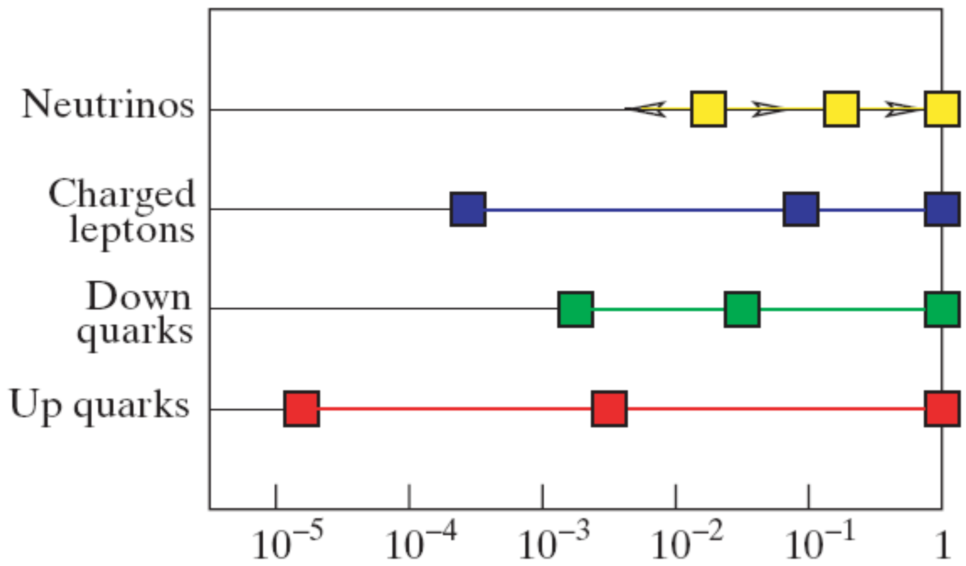
Flavour physics and CP violation informal introduction

The Standard Model Zoo

SU(3) × SU(2) × U(1) *[g; W, Z; γ]*

Pattern of the fermion masses is not currently not understood !!!



Mass hierarchies (from *hep-ph/0603118*). The heaviest fermion of a given type has unit mass.

Masses come out of interactions in the Standard Model and these interactions conserve (or do not conserve…) particular symmetries.

SM is a gauge theory, based on the symmetry group $SU(3)_C \times SU(2)_L \times U(1)_Y$

$$\left.\begin{array}{l} 8 \text{ massless gluons} \\ 1 \text{ massless photon} \\ 3 \text{ massive bosons: } W^{\pm}, Z^0 \end{array}\right\} \begin{array}{l} 12 \text{ spin-1} \\ \text{gauge fields} \end{array}$$

The fermionic matter content is given by 3-fold family structure

$$\begin{bmatrix} \nu_e & u \\ e^- & d' \end{bmatrix}, \quad \begin{bmatrix} \nu_\mu & c \\ \mu^- & s' \end{bmatrix}, \quad \begin{bmatrix} \nu_\tau & t \\ \tau^- & b' \end{bmatrix}$$

where (each quark appears in 3 different colours)

$$\begin{bmatrix} \nu_\ell & q_u \\ \ell^- & q_d \end{bmatrix} \equiv \begin{pmatrix} \nu_\ell \\ \ell^- \end{pmatrix}_L, \begin{pmatrix} q_u \\ q_d \end{pmatrix}_L, \ell^-_R, q_{uR}, q_{dR}$$

+ corresponding antiparticles

left-handed fields are $SU(2)_L$ doublets, while their right-handed partners transform as $SU(2)_L$ singlets. The 3 fermionic families appear to have identical properties (gauge interactions); they only differ by their mass and their flavour quantum number!

$$\boxed{\begin{array}{l} \text{The gauge symmetry is broken by the} \\ \text{vacuum} \to \text{triggers the Spontaneous} \\ \text{Symmetry Breaking} \\ \qquad SU(3)_C \times SU(2)_L \times U(1)_Y \xrightarrow{SSB} SU(3)_C \times U(1)_{QED} \\ \checkmark \text{ Appearance of a phys. scalar Higgs} \\ \checkmark \text{ Quark masses \& mixings also generated} \\ \quad \text{through SSB} \end{array}}$$

QED

For a free Dirac fermion

$$\mathcal{L}_0 = i\bar{\psi}(x)\gamma^\mu \partial_\mu \psi(x) - m\bar{\psi}(x)\psi(x)$$

is invariant under global $U(1)$ transformation

$$\psi(x) \xrightarrow{U(1)} \psi'(x) = \exp\{iq\theta\}\psi(x)$$

"gauge principle": $U(1)$ phase invariance should hold locally $(\theta = \theta(x))$

$$\partial_\mu \psi(x) \xrightarrow{U(1)} \exp\{iq\theta\}(\partial_\mu + iq\partial_\mu\theta)\psi(x)$$

In order to cancel $\partial_\mu\theta$ term one introduces a new spin-1 field

$$A_\mu(x) \xrightarrow{U(1)} A'_\mu(x) = A_\mu(x) + \frac{1}{e}\partial_\mu\theta$$

and defines the covariant derivative

$$D_\mu\psi(x) \equiv [\partial_\mu - ieq A_\mu(x)]\psi(x)$$

$$\hookrightarrow D_\mu\psi(x) \xrightarrow{U(1)} (D_\mu\psi)'(x) = \exp\{iq\theta\}D_\mu\psi(x)$$

Then

$$\mathcal{L} \equiv i\bar{\psi}(x)\gamma^\mu D_\mu\psi(x) - m\bar{\psi}(x)\psi(x) =$$

$$= \mathcal{L}_0 + eq A_\mu(x)\bar{\psi}(x)\gamma^\mu\psi(x)$$ is invariant under local $U(1)$ transformation

The gauge principle generated the int. between fermion and A_μ (familiar QED vertex)

(corresponding eq is completely arbitrary)

A possible mass term for the gauge field $\mathcal{L}_m = \frac{1}{2}m^2 A^\mu A_\mu$ is forbidden because it would violate gauge invariance $\Rightarrow m_\gamma = 0$!

Vector notation in colour space (q_f^α – a quark field of colour α and flavour f)

$$q_f^T \equiv (q_f^1, q_f^2, q_f^3)$$

The free Lagrangian

$$\mathcal{L}_0 = \sum_f \bar{q}_f (i\gamma^\mu \partial_\mu - m_f) q_f$$

is invariant under arbitrary global $SU(3)_C$ transformation in colour space

$$q_f^\alpha \to (q_f^\alpha)' = U^\alpha_\beta q_f^\beta, \quad UU^\dagger = U^\dagger U = 1, \quad \det U = 1$$

The $SU(3)_C$ matrices can be written in a form: $U = \exp\{i\frac{\lambda^a}{2}\theta_a\}$, where $\frac{1}{2}\lambda^a$ ($a = 1, \dots, 8$) denote 8 generators of $SU(3)_C$

The matrices λ^a are traceless and satisfy the commutation relations

$$\left[\frac{\lambda^a}{2}, \frac{\lambda^b}{2}\right] = i f^{abc} \frac{\lambda^c}{2}$$

non-abelian group!

As in the QED case, the Lagrangian is also required to be invariant under <u>local</u> $SU(3)_C$ transformations

$\theta_a = \theta_a(x)$

↳ 8 diff. gauge bosons $G_a^\mu(x)$, the so-called gluons, are needed!

$$D^\mu q_f \equiv \left[\partial^\mu - i g_s \frac{\lambda^a}{2} G_a^\mu(x)\right] q_f \equiv \left[\partial^\mu - i g_s G^\mu(x)\right] q_f$$

Introduce compact matrix notation

$$[G^\mu(x)]_{\alpha\beta} = \left(\frac{\lambda^a}{2}\right)_{\alpha\beta} G_a^\mu(x)$$

QCD-2

We want $D^\mu_{f f}$ to transform in exactly the same way as the colour vector q_f: this fixes the transf. properties of the gauge fields

$$D^\mu \to (D^\mu)' = U D^\mu U^\dagger$$

$$G^\mu \to (G^\mu)' = U G^\mu U^\dagger - \frac{i}{g_s}(\partial^\mu U)U^\dagger$$

Under infinitely small $SU(3)_c$ transformation:

$$q_f^\alpha \to (q_f^\alpha)' = q_f^\alpha + i\left(\frac{\lambda^a}{2}\right)_{\alpha\beta} \delta\theta_a q_f^\beta$$

$$G_a^\mu \to (G_a^\mu)' = G_a^\mu + \frac{1}{g_s}\partial^\mu(\delta\theta_a) - f^{abc}\delta\theta_b G_c^\mu$$

add. term involving the gluon fields themselves

$SU(3)_c$ invariant:

$$\mathcal{L}_{QCD} = \underbrace{-\frac{1}{4} G_a^{\mu\nu} G^a_{\mu\nu}} + \sum_f \bar{q}_f (i\gamma^\mu D_\mu - m_f) q_f$$

gauge-invariant kinetic term for gluon fields

$$G_a^{\mu\nu}(x) = \partial^\mu G_a^\nu - \partial^\nu G_a^\mu + g_s f^{abc} G_b^\mu G_c^\nu$$

- all interactions are given in terms of single universal coupling g_s (strong int. constant)

- New feature (not present in QED): self-interactions among the gauge fields → explain features like asymptotic freedom and confinement

Invariance properties with respect to the transformations have been always important in physics

1. *translations in* R^3
2. *rotations in* R^3
3. *time translations*

1. *momentum*
2. *angular momentum*
3. *energy*

invariance $\Rightarrow$ conservation

Gauge symmetry – invariance with respect to the transformations in «internal» space

In the SM this space has structure of U(1) × SU(2) × SU(3)

Besides continuous symmetries of prime importance in high energy physics are discrete transformations

- *C – charge conjugation*
- *P – space inversion*
- *T – time reflection*

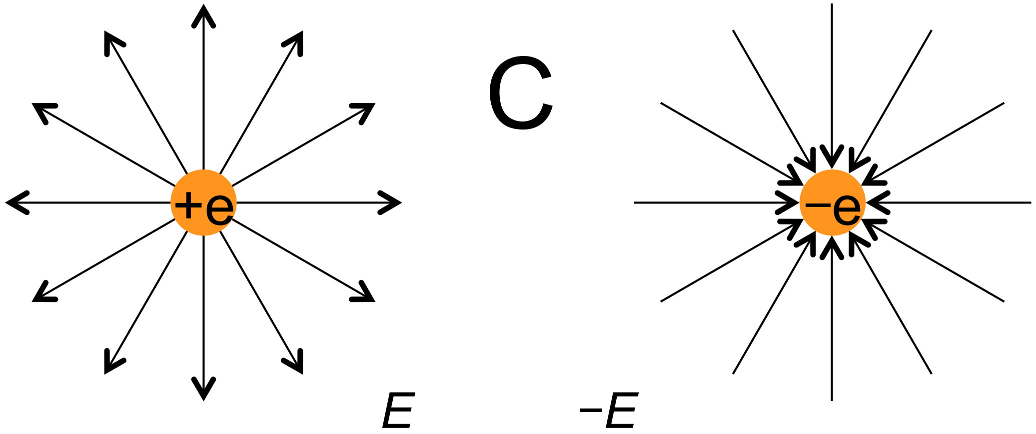
P transformation

x_i	coordinate	$x_i \rightarrow x'_i = -x_i$	
$v_i = \dfrac{dx_i}{dt}$	velocity	$v_i \rightarrow v'_i = -v_i$	
p_i	momentum	$p_i \rightarrow p'_i = -p_i$	
$L_i = \varepsilon_{ijk} x_j p_k$	angular momentum	$L_i \rightarrow L'_i = L_i$	
s_i	spin	$s_i \rightarrow s'_i = s_i$	
ρ	charge density	$\rho_i \rightarrow \rho'_i = \rho_i$	
$j_i = \rho v_i$	charge current	$j_i \rightarrow j'_i = -j_i$	
$E_i = -\dfrac{\partial \phi}{dx_i} - \dfrac{\partial A_i}{dt}$	electric field	$E_i \rightarrow E'_i = -E_i$	ϕ scaler potential
	magnetic field	$B_i \rightarrow B'_i = B_i$	$\boldsymbol{A}$ vector potential

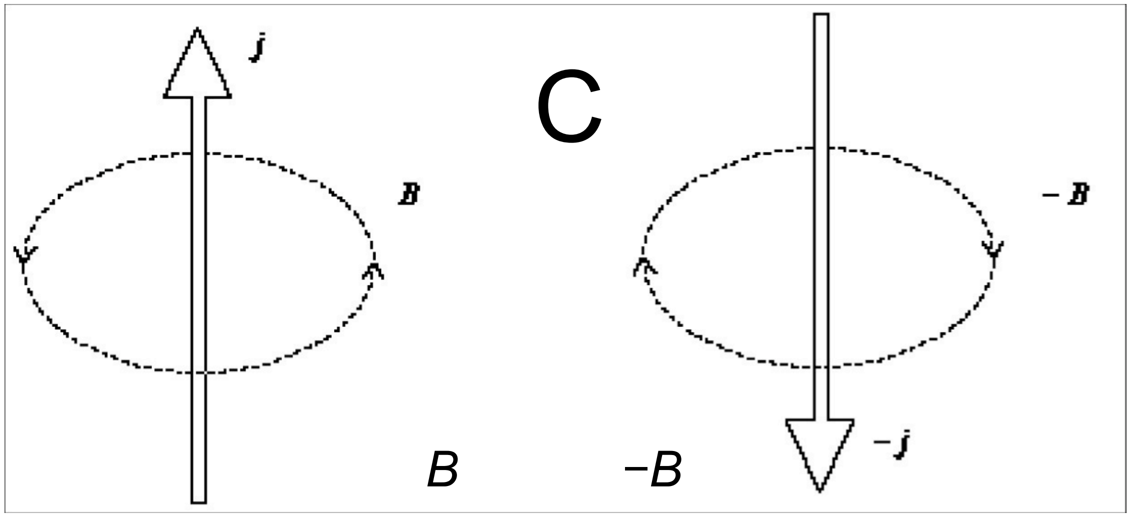
C transformation

e	electric charge	$e \to e' = -e_i$
x_i	coordinate	$x_i \to x'_i = x_i$
$v_i = \frac{dx_i}{dt}$	velocity	$v_i \to v'_i = v_i$
p_i	momentum	$p_i \to p'_i = p_i$
$L_i = \varepsilon_{ijk} x_j p_k$	angular momentum	$L_i \to L'_i = L_i$
s_i	spin	$s_i \to s'_i = s_i$
ρ	charge density	$\rho_i \to \rho'_i = -\rho_i$
$j_i = \rho v_i$	charge current	$j_i \to j'_i = -j_i$

E field



B field



C transformation

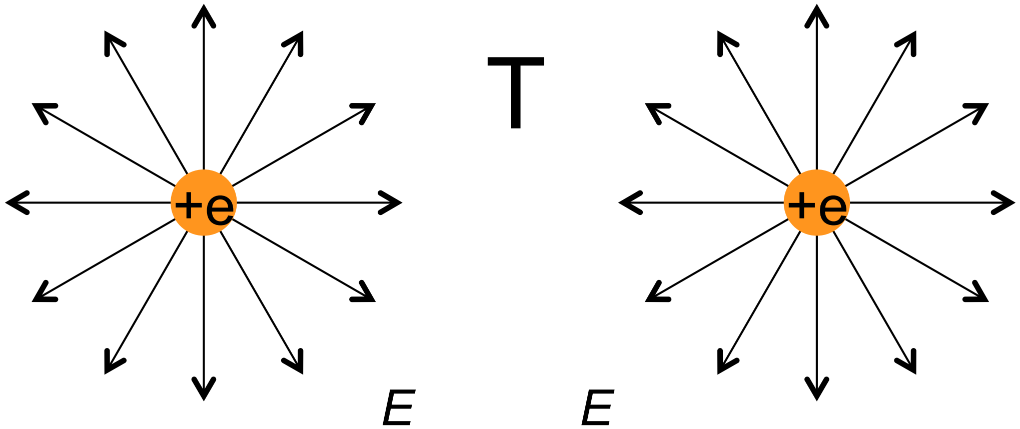
e	electric charge	$e \to e' = -e_i$
x_i	coordinate	$x_i \to x'_i = x_i$
$v_i = \dfrac{dx_i}{dt}$	velocity	$v_i \to v'_i = v_i$
p_i	momentum	$p_i \to p'_i = p_i$
$L_i = \varepsilon_{ijk} x_j p_k$	angular momentum	$L_i \to L'_i = L_i$
s_i	spin	$s_i \to s'_i = s_i$
ρ	charge density	$\rho_i \to \rho'_i = -\rho\ _i$
$j_i = \rho v_i$	charge current	$j_i \to j'_i = -j_i$
$E_i = -\dfrac{\partial \phi}{dx_i} - \dfrac{\partial A_i}{dt}$	electric field	$E_i \to E'_i = -E_i$
$B_i = \varepsilon_{ijk} \dfrac{\partial A_k}{\partial x_j}$	magnetic field	$B_i \to B'_i = -B_i$

($\phi \to \phi' = -\phi$. $\boldsymbol{A} \to \boldsymbol{A}' = -\boldsymbol{A}$)

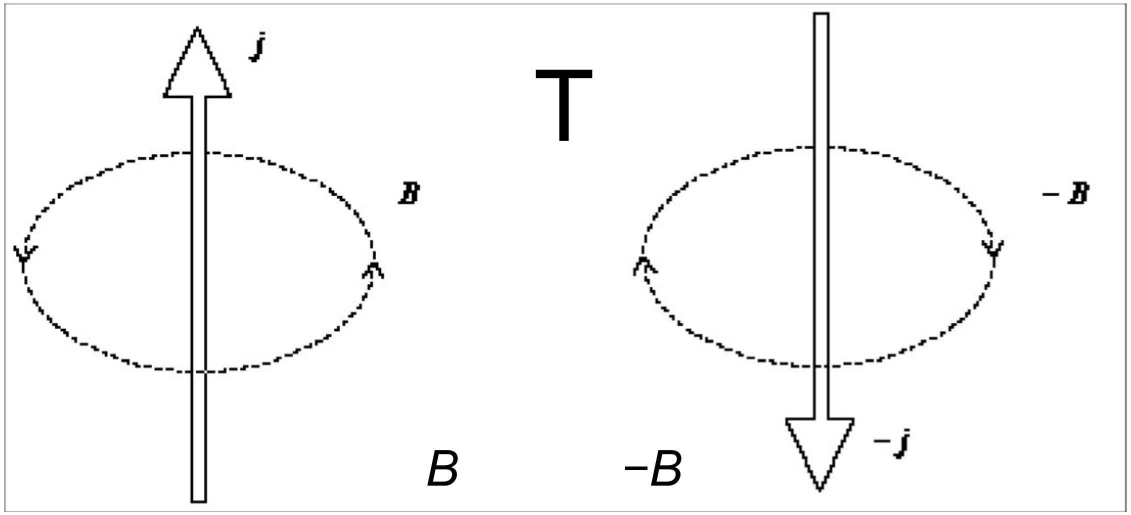
T transformation

t	time	$t \to t' = -t_i$
x_i	coordinate	$x_i \to x'_i = x_i$
$v_i = \dfrac{dx_i}{dt}$	velocity	$v_i \to v'_i = -v_i$
p_i	momentum	$p_i \to p'_i = -p_i$
$L_i = \varepsilon_{ijk} x_j p_k$	angular momentum	$L_i \to L'_i = -L_i$
s_i	spin	$s_i \to s'_i = -s_i$
ρ	charge density	$\rho_i \to \rho'_i = \rho_i$
$j_i = \rho v_i$	charge current	$j_i \to j'_i = -j_i$

E field



B field



T transformation

t	time	$t \to t' = -t_i$
x_i	coordinate	$x_i \to x'_i = x_i$
$v_i = \dfrac{dx_i}{dt}$	velocity	$v_i \to v'_i = -v_i$
p_i	momentum	$p_i \to p'_i = -p_i$
$L_i = \varepsilon_{ijk} x_j p_k$	angular momentum	$L_i \to L'_i = -L_i$
s_i	spin	$s_i \to s'_i = -s_i$
ρ	charge density	$\rho_i \to \rho'_i = \rho_i$
$j_i = \rho v_i$	charge current	$j_i \to j'_i = -j_i$
$E_i = -\dfrac{\partial\phi}{dx_i} - \dfrac{\partial A_i}{dt}$	electric field	$E_i \to E'_i = E_i$
$B_i = \varepsilon_{ijk} \dfrac{\partial A_k}{\partial x_j}$	magnetic field	$B_i \to B'_i = -B_i$

($\phi \to \phi' = \phi$. $\boldsymbol{A} \to \boldsymbol{A}' = -\boldsymbol{A}$)

Experimental fact: *strong and electromagnetic interactions are C, P, T, CP, CT, PT and CPT invariant.*

CPT theorem:

Antiparticles and their interactions are indistinguishable from particles moving along the same world-lines but in opposite directions in the (3+1) dimensional space-time.

In particular, the mass of any particle is strictly equal to the mass of its antiparticle (experimentally checked in 1 part to 10^{18} in K-meson studies). The same is true for lifetimes. Magnetic moments of particle and antiparticle are opposite in sign

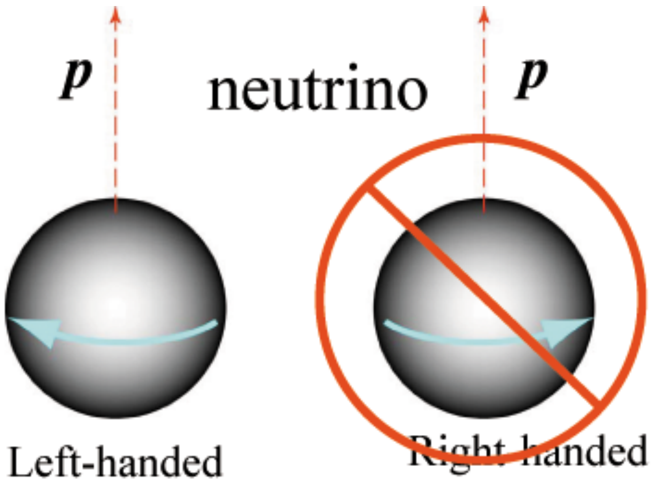
The SM strictly conserves CPT. There are no however any theoretical reason why C, P and T should be conserved separately.

Often in physics if something can happen – it does happen !

Weak interactions violate P-parity

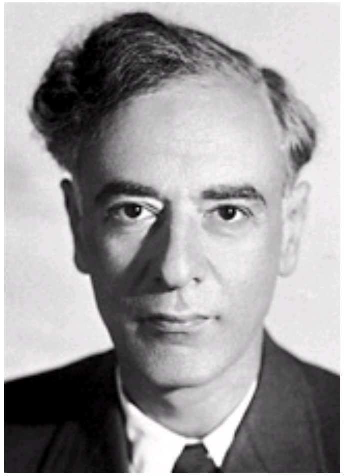
T.D.Lee, C.N.Yang, 1956

C.S.Wu, 1957

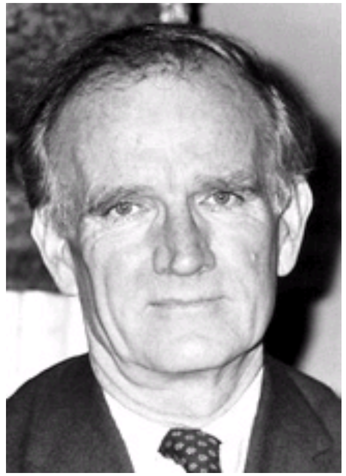


100% C-violation in neutrino sector

$$
\begin{tikzcd}
\pi^+ \to \mu^+_R + \nu_L \arrow[rr, "P"] \arrow[d, "C"] \arrow[drr, "CP"] & & \pi^+ \to \mu^+_L + \cancel{\nu_R} \\
\pi^- \to \mu^-_R + \cancel{\bar{\nu}_L} & & \pi^- \to \mu^-_L + \bar{\nu}_R
\end{tikzcd}
$$



L.D.Landau, 1959: hypothesis of combined CP-parity conservation



J.Cronin, V.Fitch, 1964: CP-violation discovery in neutral K-mesons decays.

assuming CP-invariance of the weak interaction the decays of K^0 were thought are described by the eigenvalues of the CP-operator

$$CP\,|K^0\rangle \rightarrow |\bar{K}^0\rangle$$
$$CP\,|\bar{K}^0\rangle \rightarrow |K^0\rangle$$

identical to the action of the C-operator alone since K^0 spin is zero

If
$$|K_1\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right)$$
$$|K_2\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right),$$

then

$$CP\,|K_1\rangle = \frac{1}{\sqrt{2}}\left(|\bar{K}^0\rangle + |K^0\rangle\right) = |K_1\rangle$$
$$CP\,|K_2\rangle = \frac{1}{\sqrt{2}}\left(|\bar{K}^0\rangle - |K^0\rangle\right) = -|K_2\rangle$$

$|K_1\rangle$ and $|K_2\rangle$ are the eigenstates of the CP-operator with the eigenvalues of +1 and -1 repectively

$|K^0\rangle$ and $|\bar{K}^0\rangle$ states are not distinguishable by their decays $\Longrightarrow$ $|K_1\rangle$ and $|K_2\rangle$ do decay differently!

Final states of neutral kaon decays

pionic final states : $\pi\pi$ *and* $\pi\pi\pi$

$$\mathcal{CP}|\,\pi^+\pi^-\rangle = +|\,\pi^+\pi^-\rangle \;\Rightarrow\; K_1 \to \pi^+\pi^-$$
$$\mathcal{CP}|\,\pi^0\pi^0\rangle = +|\,\pi^0\pi^0\rangle \;\Rightarrow\; K_1 \to \pi^0\pi^0$$

$$\mathcal{CP}|\,\pi^+\pi^-\pi^0\rangle = \begin{array}{ll} +|\,\pi^+\pi^-\pi^0\rangle_{l_{\pi^+\pi^-}=1} & \Rightarrow K_1 \\ -|\,\pi^+\pi^-\pi^0\rangle_{l_{\pi^+\pi^-}=0} & \Rightarrow K_2 \end{array} \to \pi^+\pi^-\pi^0$$
$$\mathcal{CP}|\,\pi^0\pi^0\pi^0\rangle = -|\,\pi^0\pi^0\pi^0\rangle \;\Rightarrow\; K_2 \to \pi^0\pi^0\pi^0$$

By detecting a pionic final state one can tag the $\mathcal{CP}$ symmetry of neutral kaons at the time of their decay, assuming no direct $\mathcal{CP}$-violation.

Measured properties of neutral kaons

(PDG98)

Final State	K_S Branching Ratio	K_L Branching Ratio
$\pi^+\pi^-$	$(68.61 \pm 0.28)\%$	$(2.067 \pm 0.035) \times 10^{-3}$
$\pi^0\pi^0$	$(31.39 \pm 0.28)\%$	$(9.36 \pm 0.20) \times 10^{-4}$
$\pi^+\pi^-\pi^0$	$(3.4 + 1.1 - 0.9) \times 10^{-7}$	$(12.56 \pm 0.20)\%$
$\pi^0\pi^0\pi^0$	$< 1.9 \times 10^{-5}$ (90% CL)	$(21.12 \pm 0.27)\%$
$\pi e \nu$	$(6.70 \pm 0.07) \times 10^{-4}$ †	$(38.78 \pm 0.27)\%$
$\pi \mu \nu$	$(4.69 \pm 0.06) \times 10^{-4}$ †	$(27.17 \pm 0.25)\%$

† : Calculated from K_L semileptonic rates and the K_S lifetime assuming $\Delta S = \Delta Q$.

K_S and K_L are no eigenstates of $\mathcal{CP}$:

$$|K_S\rangle \propto |K_1\rangle + \varepsilon_S |K_2\rangle \; ; \; |K_L\rangle \propto |K_2\rangle + \varepsilon_L |K_1\rangle$$

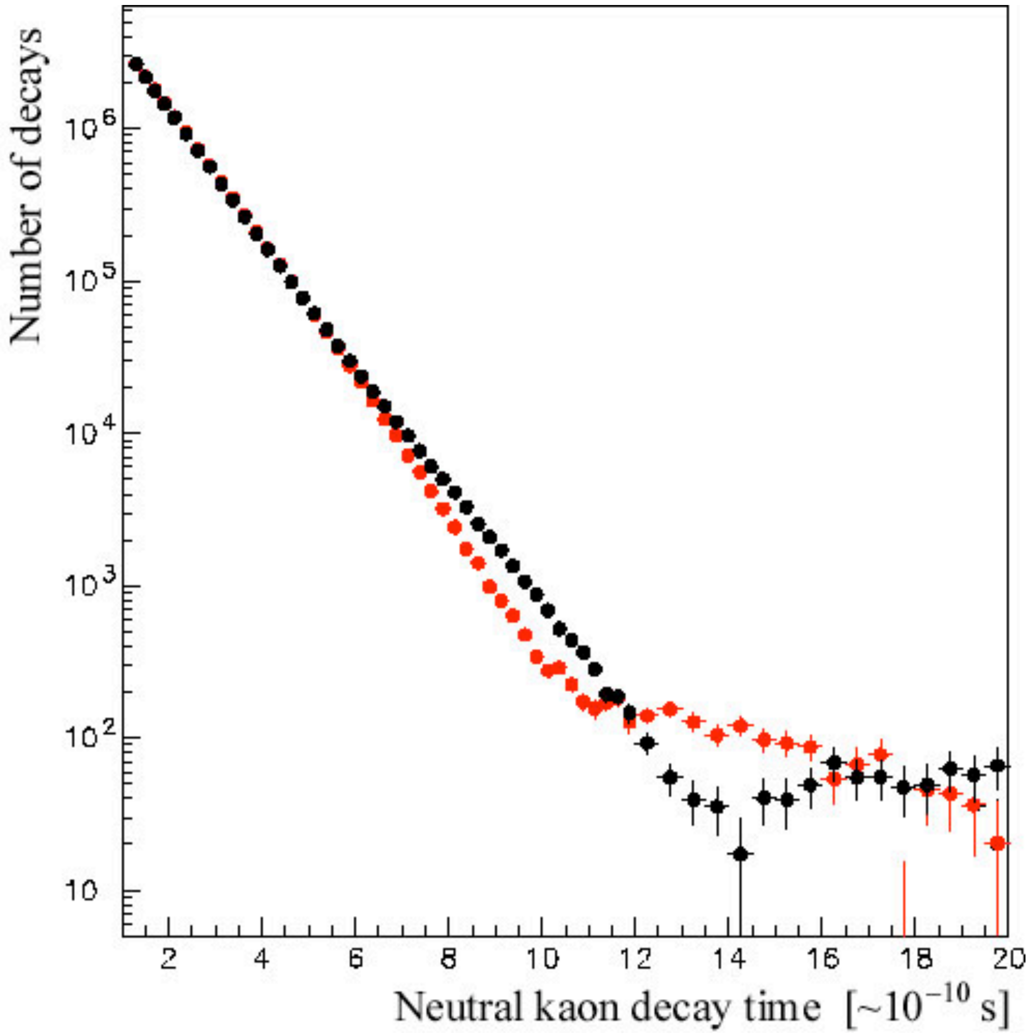
Mean Life:

$$\tau_S = (0.8934 \pm 0.0008) \times 10^{-10} s$$
$$\tau_L = (517 \pm 4) \times 10^{-10} s$$

Mass:

$$m_L - m_S = (0.5301 \pm 0.0014) \times 10^{10} \hbar s^{-1}$$

In the world of elementary particles:
(CPLEAR 1999)



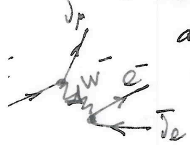
neutral kaon
decay time distribution
$\neq$
anti-neutral kaon
decay time distribution

CP violation

Interactions of fermions: charge currents

Charge currents

The interactions of quarks and leptons with $W^\pm$



- Only left-handed fermions and right-handed antifermions couple to $W^\pm$
  $\Rightarrow$ 100% breaking of parity and charge conjugation

- $W^\pm$ couple to the fermionic doublets, where the electric charges of the two fermion partners differ in one unit

$$W^- \to e^-\bar\nu_e,\ \mu^-\bar\nu_\mu,\ \tau^-\bar\nu_\tau,\ d'\bar u,\ s'\bar c$$

On-shell production of the top quark through $W^- \to b'\bar t$ is kinematically forbidden

- All fermion doublets couple to $W^\pm$ with the same universal strength

- The doublet partners of u, c and t quarks appear to be mixtures of the three charge $-1/3$ quarks

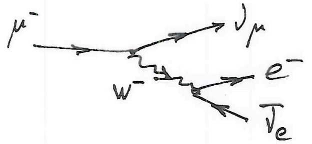
$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V \begin{pmatrix} d \\ s \\ b \end{pmatrix} \qquad VV^+ = V^+V = 1$$

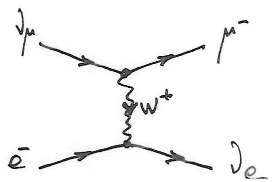
The mass eigenstates d, s, b are different from the weak eigenstates d', s', b'. They are related through 3×3 unitary matrix V

- ν_e, ν_μ, ν_τ are also mixtures of mass eigenstates

$$m^2_{\nu_3} - m^2_{\nu_2} \sim 3\times 10^{-3}\,eV^2$$

$$m^2_{\nu_2} - m^2_{\nu_1} \sim 8\times 10^{-5}\,eV^2$$

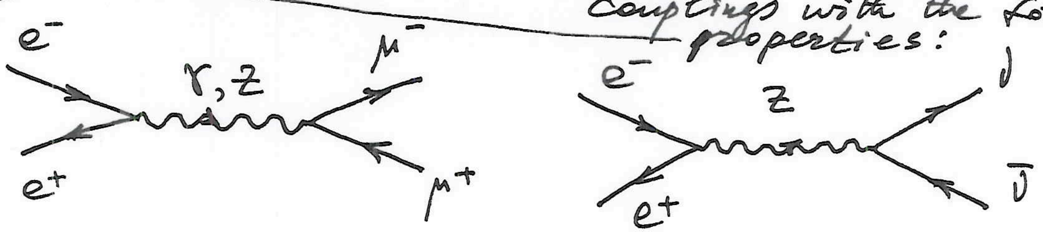




Interactions of fermions: neutral currents

neutral Currents

neutral currents of the em and weak int. have fermionic couplings with the following properties:



- all int. vertices are flavour conserving
  $\gamma f\bar f$, $Z f\bar f$.
  $\mu \not\to e\gamma$ or $Z \not\to e^\pm\mu^\mp$ not observed

- Interactions depend on the fermion electric charge Q_f. Fermions with the same Q_f have exactly the same universal couplings. Neutrinos do not have em int. ($Q_\nu = 0$), but they have a non-zero coupling to Z

- Photons have the same interaction for both fermion chiralities, but Z couple differently to left-h and right-h fermions. The ν coupling to Z involves only left-handed chiralities

- There are three different light neutrino species

$SU(2)_L \times U(1)_Y$ theory

To describe weak int. $\to$ $\mathcal{L}$ with several fermionic flavours and different properties for left- and right-handed fields

- Left-h fermions should appear in doublets
- Massive $W^\pm$ and Z !

$\hookrightarrow$ Consider $G \equiv SU(2)_L \times U(1)_Y$

(for simplicity . single family of q and leptons

$$\psi_1(x) = \begin{pmatrix} u \\ d \end{pmatrix}_L , \quad \psi_2(x) = u_R \quad \psi_3(x) = d_R$$

Our discussion is valid as well for

$$\psi_1(x) = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L , \quad \psi_2(x) = \nu_{eR} , \quad \psi_3(x) = e^-_R$$

Free Lagrangian: $\mathcal{L}_0 = \sum_{j=1}^{3} i \bar{\psi}_j(x) \gamma^\mu \partial_\mu \psi_j(x)$

is invariant under global

G transformation in flavour space:

$$\psi_1(x) \xrightarrow{G} \psi_1'(x) \equiv \exp\{ i y_1 \beta \} U_L \psi_1(x)$$
$$\psi_2(x) \xrightarrow{G} \psi_2'(x) \equiv \exp\{ i y_2 \beta \} \psi_2(x)$$
$$\psi_3(x) \xrightarrow{G} \psi_3'(x) = \exp\{ i y_3 \beta \} \psi_3(x)$$

where $SU(2)_L$ transformation $U_L = \exp\{ i \frac{\sigma_i}{2} \alpha^i \}$ only acts on the doublet field $\psi_1(x)$

The parameters y_i are called hypercharges in analogy to QED

The mass term is not included to $\mathcal{L}_0$ because it would mix l- and r-handed fields spoiling $SU(2)_L$

Require local invariance under $SU(2)_L \times U(1)_Y$

Since we have 4 gauge parameters $\alpha_i(x)$ and $\beta(x)$, 4 different gauge bosons are needed!

$$D_\mu \psi_1(x) \equiv [\partial_\mu - i g \widetilde{W}_\mu(x) - i g' y_1 B_\mu(x)] \psi_1(x)$$
$$D_\mu \psi_2(x) \equiv [\partial_\mu - i g' y_2 B_\mu(x)] \psi_2(x)$$
$$D_\mu \psi_3(x) \equiv [\partial_\mu - i g' y_3 B_\mu(x)] \psi_3(x)$$

where. $\widetilde{W}_\mu(x) = \frac{\sigma_i}{2} W^i_\mu(x)$

$D_\mu \psi_j(x)$ transforms exactly as $\psi_j(x)$

This fixes transf. properties of the gauge fields

$$B_\mu(x) \xrightarrow{G} B'_\mu(x) \equiv B_\mu(x) + \frac{1}{g'}\partial_\mu \beta(x)$$

$$\widetilde{W}_\mu(x) \xrightarrow{G} \widetilde{W}'_\mu \equiv U_L(x)\,\widetilde{W}_\mu\, U_L^\dagger(x) - \frac{i}{g}\partial_\mu U_L(x)\, U_L^\dagger(x)$$

The Lagrangian

$$\mathcal{L} = \sum_{j=1}^{3} i\,\bar{\psi}_j(x)\,\gamma^\mu D_\mu \psi_j(x)$$

is invariant under local G transformation

The gauge symmetry forbids to write a mass term for the gauge bosons. Fermionic masses are also not possible, because they would communicate the left- and right-handed fields with different transformation properties → produce an explicit breaking of the gauge symmetry

↪ $SU(2)_L \times U(1)_Y$ $\mathcal{L}$ only contains massless fields a fermionic mass term not allowed

$$\mathcal{L}_m = -m\bar{\psi}\psi = -m(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L)$$

However since we have introduced an additional scalar doublet into the model, one can write the following gauge-invariant fermion-scalar coupling

$$\mathcal{L}_Y = c_1 (\bar{u},\bar{d})_L \begin{pmatrix}\phi^{(+)} \\ \phi^{(0)}\end{pmatrix} d_R + c_2 \overbrace{}^{(\bar{u},\bar{d})_L} \begin{pmatrix}\phi^{(0)*} \\ -\phi^{(-)}\end{pmatrix} u_R +$$

$$+ c_3 (\bar{\nu}_e \bar{e})_L \begin{pmatrix}\phi^{(+)} \\ \phi^{(0)}\end{pmatrix} e_R + h.c.$$

In the unitary gauge (after SSB), this Yukawa-type $\mathcal{L}$ takes the simpler form:

$$\mathcal{L}_Y = \frac{1}{\sqrt{2}}(v+H)\{c_1\bar{d}d + c_2\bar{u}u + c_3\bar{e}e\}$$

$\hookrightarrow$ SSB mechanism generates also fermion masses:

$$m_d = -c_1\frac{v}{\sqrt{2}}, \quad m_u = -c_2\frac{v}{\sqrt{2}},$$

$$m_e = -c_3\frac{v}{\sqrt{2}}$$

N_G generations of fermions

$\nu_j, \ell'_j, u'_j, d'_j$ - members of the weak family with definite transf. properties under the gauge group

$\hookrightarrow$ Most general Yukawa Lagrangian:

$$\mathcal{L}_Y = \sum_{jk} \left\{ (\bar{u}'_j \; \bar{d}'_j)_L \left[c^{(d)}_{jk} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} d'_{kR} + c^{(u)}_{jk} \begin{pmatrix} \phi^{(0)*} \\ -\phi^{(-)} \end{pmatrix} u'_{kR} \right] + (\bar{\nu}'_j, \bar{\ell}'_j)\, c^{(\ell)}_{jk} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} \ell'_{kR} \right\} + \text{h.c.}$$

$c^{(d)}_{jk}, c^{(u)}_{jk}, c^{(\ell)}_{jk}$ — arbitrary coupling constants

After SSB

$$\mathcal{L}_Y = -\left(1+\frac{H}{v}\right)\left\{ \bar{d}'_L M'_d d'_R + \bar{u}'_L M'_u u'_R + \bar{\ell}'_L M'_\ell \ell'_R + \text{h.c.} \right\}$$

where d', u', ℓ' - vectors in N_G-dimensional flavour space

The corresponding mass matrices are given by

$$(M'_d)_{ij} \equiv -c^{(d)}_{ij} \frac{v}{\sqrt{2}}, \quad (M'_u)_{ij} \equiv -c^{(u)}_{ij} \frac{v}{\sqrt{2}}$$

$$(M'_\ell)_{ij} \equiv -c^{(\ell)}_{ij} \frac{v}{\sqrt{2}}$$

The diagonalization of these mass matrices determines the mass eigenstates d_j, u_j and ℓ_j, which are linear combinations of the corresponding weak eigenstates d'_j, u'_j and ℓ'_j

---

M'_d can be decomposed as $M'_d = H_d U_d = S^\dagger_d \mathcal{M}_d S_d U_d$

$M'_u = H_u U_u = S^\dagger_u \mathcal{M}_u S_u U_u$

$\mathcal{M}_d = \text{diag}(m_d, m_s, m_b, \dots)$, $\mathcal{M}_u = \text{diag}(m_u, m_c, m_t, \dots)$

$\mathcal{M}_\ell = \text{diag}(m_e, m_\mu, m_\tau, \dots)$

$\hookrightarrow$ In terms of diagonal mass matrices

$$\mathcal{L}_Y = -\left(1+\frac{H}{v}\right)\left\{ \bar{d}_L \mathcal{M}_d d_R + \bar{u}_L \mathcal{M}_u u_R + \bar{\ell}_L \mathcal{M}_\ell \ell_R \right\}$$

The mass eigenstates

$$d_L = S_d d'_L, \quad u_L \equiv S_u u'_L, \quad \ell_L \equiv S_\ell \ell'_L$$

$$d_R \equiv S_d U_d d'_R, \quad u_R \equiv S_u U_u u'_R, \quad \ell_R \equiv S_\ell U_\ell \ell'_R$$

Note: Higgs coupling are proportional to the corresponding fermion masses

Since $\bar{f}'_L f'_L = \bar{f}_L f_L$ ($f = u, d, \ell$) the form of the neutral current part of the $SU(2)_L \times U(1)_Y$ Lagr. does not change when expressed in terms of mass eigenstates

$\Rightarrow$ There are no FCNC in SM

On the other hand

$$\bar{u}'_L d'_L = \bar{u}_L S_u S_d^\dagger d_L \equiv \bar{u}_L V d_L$$

in general, $S_u \neq S_d$ $\Rightarrow$ If one writes the weak eigenstates in terms of mass eigenstates, a $N_G \times N_G$ matrix V (CKM matrix) appears in the quark charge current sector:

$$\mathcal{L}_{CC} = \frac{g}{2\sqrt{2}} \left\{ W_\mu^\dagger \left[\sum_{ij} \bar{u}_i \gamma^\mu (1-\gamma_5) V_{ij} d_j + \right.\right.$$

$$+ \sum_i \bar{\nu}_L \gamma^\mu$$

(V couples any "up-type" quark with all "down-type" quarks)

For the massless ν one can always redefine the ν flavours in such a way to eliminate the analogous mixing in the lepton sector: $\bar{\nu}'_L \ell'_L = \bar{\nu}'_L S^\dagger \ell_L \equiv \bar{\nu}_L \ell_L$

$\hookrightarrow$ lepton flavour conservation in the minimal SM without right-handed neutrinos

if sterile ν_R are included in the model $\to$ additional Yukawa term giving rise to neutrino mixing matrix

$(M'_\nu)_{ij} = - c_{ij}^{(\nu)} v/\sqrt{2}$

Standard Model and CP violation

Strong interaction — gluons

Electromagnetic interaction — photons

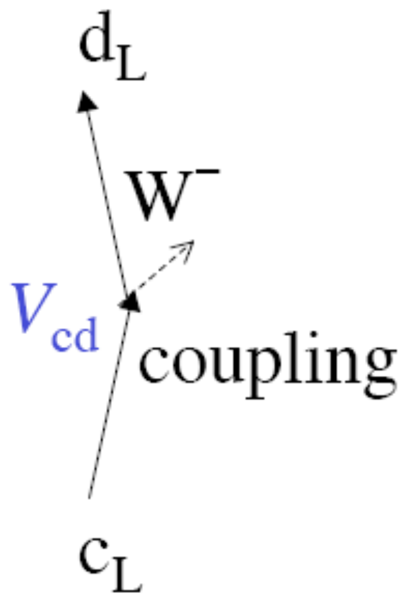
Weak interaction

neutral current

charged current: $W^{\pm}$

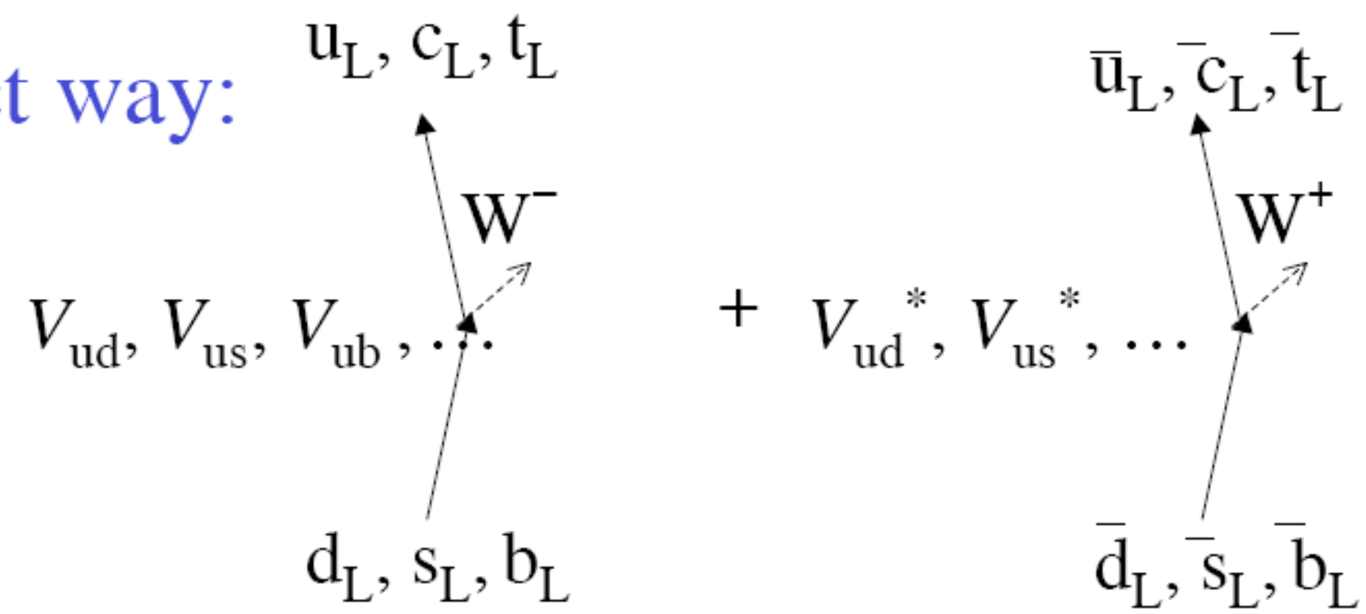
Up type quark spinor field $Q = 2/3$	Down type quark spinor field $Q = -1/3$
$\boldsymbol{U} = \begin{pmatrix} u \\ c \\ t \end{pmatrix}$	$\boldsymbol{D} = \begin{pmatrix} d \\ s \\ b \end{pmatrix}$

example



there are $3 \times 3 = 9$ V's

Correct way:



$$L \propto V_{ij}\,\overline{U}_i\,\gamma^\mu(1-\gamma_5)\,D_j\,W_\mu^\dagger + V_{ij}^*\,\overline{D}_i\,\gamma^\nu(1-\gamma_5)\,U_j\,W_\nu$$

CP conjugation

$$L_{CP} \propto V_{ij}\,\overline{D}_i\,\gamma^\mu(1-\gamma_5)\,U_j\,W_\mu + V_{ij}^*\,\overline{U}_i\,\gamma^\nu(1-\gamma_5)\,D_j\,W_\nu^\dagger$$

If $V_{ij}^* = V_{ij} \rightarrow L = L_{CP}$: i.e. CP conservation

Let us look at now: $V_{ij}\,\overline{U}_i\,\gamma^{\mu}(1-\gamma_5)\,D_j$

One family

$$\boldsymbol{V}\quad \begin{matrix}\text{1 free phase}\\ \text{1 free modula}\end{matrix} \qquad = |V|\,e^{\,i\phi}$$

$$|V|\,e^{\,i\phi}\,\bar{u}\,\gamma^{\mu}(1-\gamma_5)\,d \longrightarrow |V|\,\bar{u}\,\gamma^{\mu}(1-\gamma_5)\,d$$

Changing u quark phase: $u \rightarrow u\,e^{\,i\phi}$

Unitarity: $\boldsymbol{V}^{\dagger}\boldsymbol{V} = \boldsymbol{V}\boldsymbol{V}^{\dagger} = \boldsymbol{E}$ (one constraint)

$$|V|^2 = 1$$

0 free phase
0 free modula

NO $\cancel{\text{C}}$P

Two families $\boldsymbol{V} = \begin{pmatrix} V_{\mathrm{ud}} & V_{\mathrm{us}} \\ V_{\mathrm{cd}} & V_{\mathrm{cs}} \end{pmatrix}$ 4 free phase
4 free moduli (or rotation angles)

1) The phase of V_{ij} can be absorbed by adjusting the phase differences between i- and j- quark

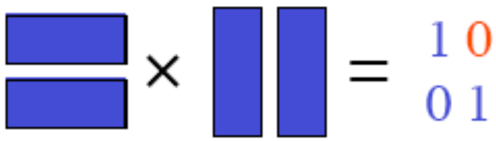
4 quarks = 3 phase differences

4 – 3 = 1 phase left

2) Unitarity $\boldsymbol{V}^\dagger \boldsymbol{V} = \boldsymbol{V}\boldsymbol{V}^\dagger = \boldsymbol{E}$: four constraints:

1 off-diagonal constraint for the phase

1 – 1 = 0 phase left



three constraint for the rest

4 – 3 = 1 rotation angle left

V **is real, i.e. no ~~CP~~.**

Explicit demonstration

$$|V_{ud}|e^{i\phi_{ud}}\,\overline{u}\,\gamma^{\mu}(1-\gamma_5)\,d + |V_{us}|e^{i\phi_{us}}\,\overline{u}\,\gamma^{\mu}(1-\gamma_5)\,s$$
$$+\,|V_{cd}|e^{i\phi_{cd}}\,\overline{c}\,\gamma^{\mu}(1-\gamma_5)\,d + |V_{cs}|e^{i\phi_{cs}}\,\overline{c}\,\gamma^{\mu}(1-\gamma_5)\,s$$

$$u \rightarrow u\,e^{i\phi_{ud}}$$

$$|V_{ud}|\,\overline{u}\,\gamma^{\mu}(1-\gamma_5)\,d + |V_{us}|e^{i(\phi_{us}-\phi_{ud})}\,\overline{u}\,\gamma^{\mu}(1-\gamma_5)\,s$$
$$+\,|V_{cd}|e^{i\phi_{cd}}\,\overline{c}\,\gamma^{\mu}(1-\gamma_5)\,d + |V_{cs}|e^{i\phi_{cs}}\,\overline{c}\,\gamma^{\mu}(1-\gamma_5)\,s$$

$$s \rightarrow s\,e^{-i(\phi_{us}-\phi_{ud})},\quad c \rightarrow c\,e^{i(\phi_{cs}-\phi_{us}+\phi_{ud})}$$

$$|V_{ud}|\,\overline{u}\,\gamma^{\mu}(1-\gamma_5)\,d + |V_{us}|\,\overline{u}\,\gamma^{\mu}(1-\gamma_5)\,s$$
$$+\,|V_{cd}|e^{i\delta}\,\overline{c}\,\gamma^{\mu}(1-\gamma_5)\,d + |V_{cs}|\,\overline{c}\,\gamma^{\mu}(1-\gamma_5)\,s$$

Out of four quark, three quark phases can be adjusted:

4 free phase $\rightarrow$ 1 free phase

Unitarity: $\boldsymbol{V}^{\dagger}\boldsymbol{V} = \boldsymbol{V}\boldsymbol{V}^{\dagger} = \boldsymbol{E}$ (4 constraints) $\begin{pmatrix} V_{ud}^{*} & V_{cd}^{*} \\ V_{us}^{*} & V_{cs}^{*} \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} \\ V_{cd} & V_{cs} \end{pmatrix}$

$$V_{ud}^{*} V_{us} + V_{cd}^{*} V_{cs} = 0 \rightarrow |V_{ud}|\,|V_{us}| + |V_{cd}|\,|V_{cs}|\, e^{-i\delta} = 0$$

0 free phase: $\delta = \pi$

$$|V_{ud}|^2 + |V_{cd}|^2 = 1,\ |V_{us}|^2 + |V_{cs}|^2 = 1$$

1 free modula or rotation angle

$$|V_{11}| = \cos\theta,\ |V_{22}| = \cos\theta,\ |V_{12}| = \sin\theta,\ |V_{21}| = \sin\theta$$

$$\boldsymbol{V} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

One rotation angle without phase: $\rightarrow$ **NO $\cancel{\mathrm{CP}}$**

(Cabibbo angle)

Three families $\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$ 9 free phase
9 free moduli (or rotation angles)

Out of six quark, five quark phases can be adjusted:

9 free phase $\rightarrow$ 4 free phase

1 0 0
0 1 0
0 0 1

Out of nine unitarity constraints, three are for the phases

4 free phase $\rightarrow$ **1 free phase**

the rest (six) are for the rotation angles

9 free rotation angles $\rightarrow$ 3 free rotation angles

Three rotation angles with one phase:

$\rightarrow$ **$\cancel{CP}$ can be generated**

Electroweak theory with 3 families can naturally accommodate CP violation in the charged current induced interactions through the complex Cabibbo-Kobayashi-Maskawa quark mixing matrix $\boldsymbol{V}$, with 4 parameters (three angles and **one phase**).

M. Kobayashi, T.Maskawa, 1974:
theoretical mechanism for
CP-violation in the SM

Idea: nontrivial superposition of non-interacting particles forms flavor eigenstate that interacts weakly

In other words: it is impossible to diagonalize simultaneously the mass term and charged currents interaction term:

$$L_{\text{int}} = \frac{g_2}{\sqrt{2}}\left(\bar{u}_L,\ \bar{c}_L,\ \bar{t}_L\right)\gamma^{\mu}\hat{V}_{CKM}\begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} W_{\mu}^{+} + h.c.$$

CKM mechanism in the SM to describe Flavour Physics

CKM matrix can be parameterized by four parameters in many different ways. The so called «Wolfenstein parameterization» is based on expansion in powers of $\lambda = |V_{us}| + O(\lambda^7) = 0.2272 \pm 0.0010$

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{\mathrm{ud}} & V_{\mathrm{us}} & V_{\mathrm{ub}} \\ V_{\mathrm{cd}} & V_{\mathrm{cs}} & V_{\mathrm{cb}} \\ V_{\mathrm{td}} & V_{\mathrm{ts}} & V_{\mathrm{tb}} \end{pmatrix}$$

$V_{\mathrm{ud}}, V_{\mathrm{us}}, V_{\mathrm{cs}}, V_{\mathrm{cb}}, V_{\mathrm{tb}}$ are (practically) real
$V_{\mathrm{cd}}, V_{\mathrm{ts}}$ could be slightly complex
$V_{\mathrm{td}}, V_{\mathrm{ub}}$ could be complex

$$\begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda\left[1 + A^2\lambda^4(\rho + i\eta)\right] & \left(1-\frac{\lambda^2}{2}\right) & A\lambda^2 \\ A\lambda^3\left[1-(\rho+i\eta)\left(1-\frac{\lambda^2}{2}\right)\right] & -A\lambda^2\left[\left(1-\frac{\lambda^2}{2}\right)+\lambda^2(\rho+i\eta)\right] & 1 \end{pmatrix}$$

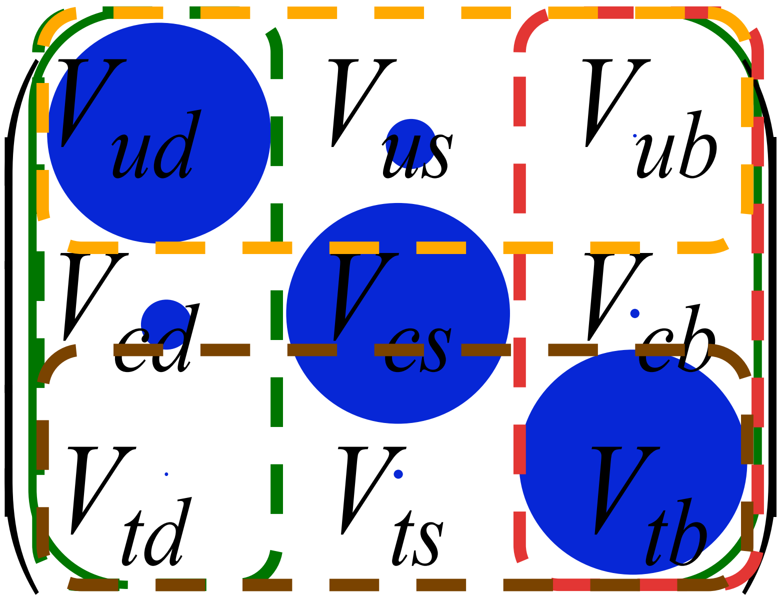
It is convenient to discuss the properties of CKM matrix in parameterization-invariant terms. Such invariant are absolute values of the matrix elements and «angles» between them

$$\eta = \arg\left(\frac{V_{ij}V_{ik}^*}{V_{lj}V_{lk}^*}\right)$$

If any of these angles is different from zero, it means that there is a complex phase in CKM matrix which cannot be rotated away. This violates CP.

«Jarlskog invariant» $$J = \left|\mathrm{Im}\ V_{i\alpha}V_{j\beta}V_{i\beta}^*V_{j\alpha}^*\right| \sim 3\times 10^{-5}$$

Off-diagonal unitarity conditions can be represented as triangles on complex plane.



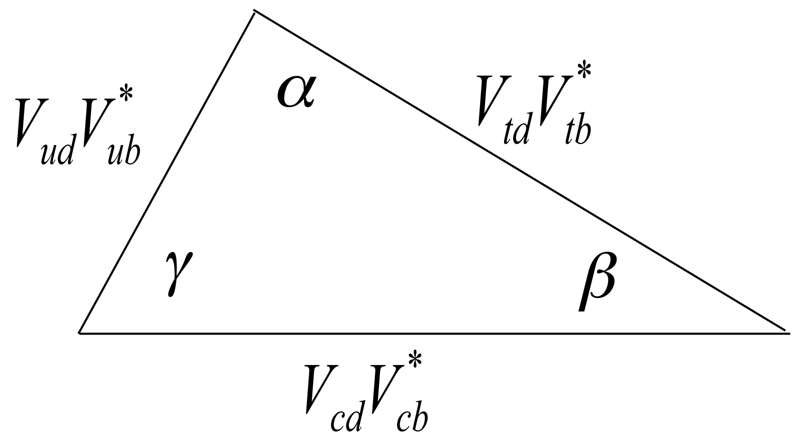
$$\boxed{V_{ud}} \cdot \boxed{V_{ub}^*} + \boxed{V_{cd}} \cdot \boxed{V_{cb}^*} + \boxed{V_{td}} \cdot \boxed{V_{tb}^*} = 0$$

$$\boxed{V_{ud}} \cdot \boxed{V_{td}^*} + \boxed{V_{us}} \cdot \boxed{V_{ts}^*} + \boxed{V_{ub}} \cdot \boxed{V_{tb}^*} = 0$$

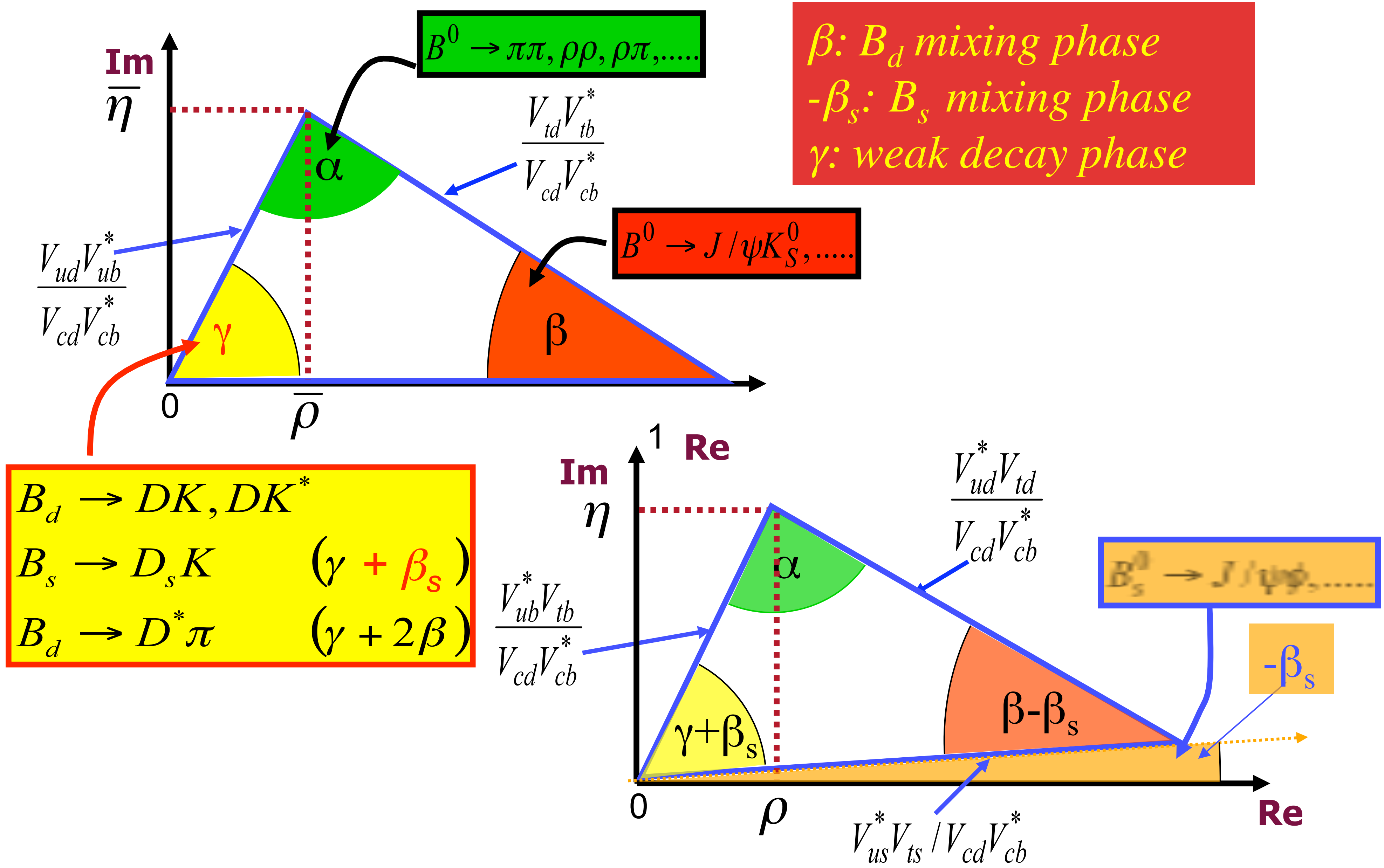
All 6 unitarity triangles have equal area but only two of them are not degenerate.

B-mesons decays are very sensitive to CP !

The Unitarity triangle:



The Unitarity Triangle

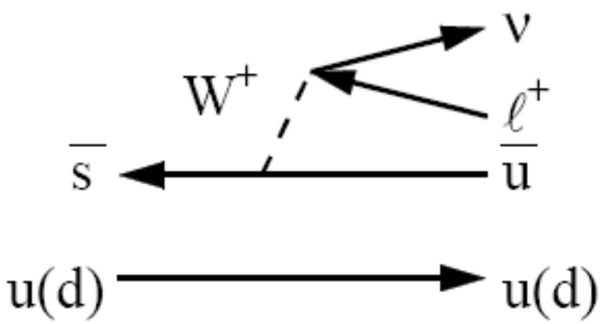


Search for New Physics in Flavour Physics

(using CP-violation and Rare Decays)

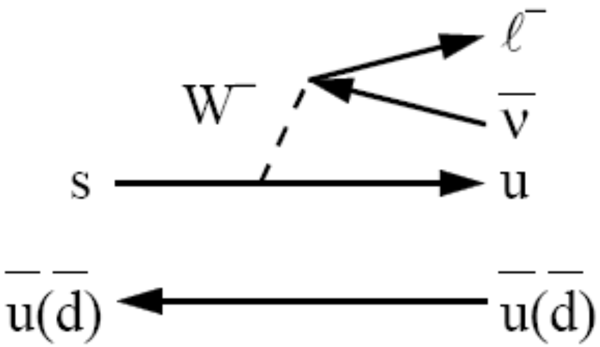
Examples of diagrams in kaon decays

semileptonic decays: with tree diagrams



$$K^+(K^0) \rightarrow \ell^+ \nu + \text{hadrons}$$

$s = +1$	0	$\Delta s = -1$
$Q = +1\ (0)$	$0\ (-1)$	$\Delta Q = -1$



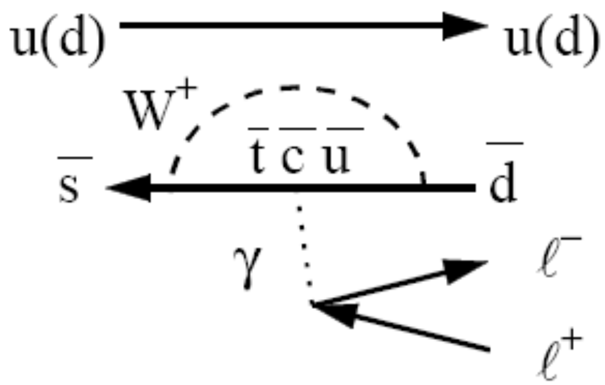
$$K^-(\overline{K^0}) \rightarrow \ell^- \overline{\nu} + \text{hadrons}$$

$s = -1$	0	$\Delta s = +1$
$Q = -1\ (0)$	$0\ (+1)$	$\Delta Q = +1$

$$\Delta Q = \Delta s \text{ rule}$$

another type of semileptonic decays: with penguin diagrams

for example

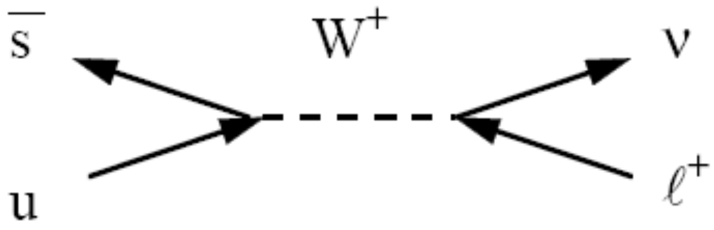


$K^+(K^0) \rightarrow \ell^+ \ell^- +$ hadrons

$+$ c.c.

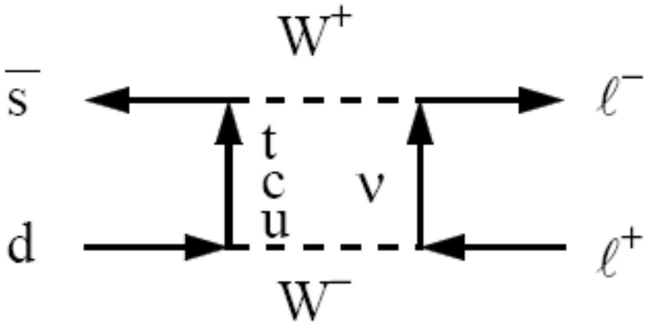
leptonic decays:

1st order electroweak interactions

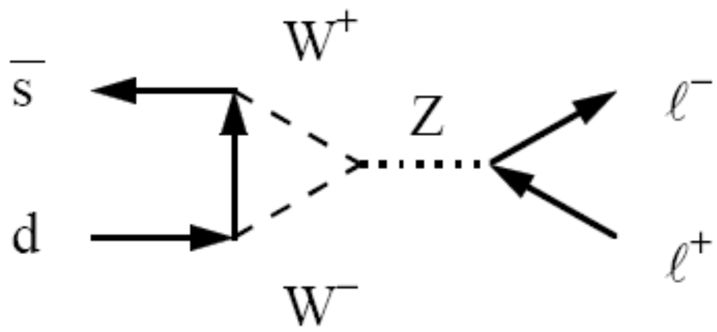


W-annihilation diagram

2nd order electroweak interactions

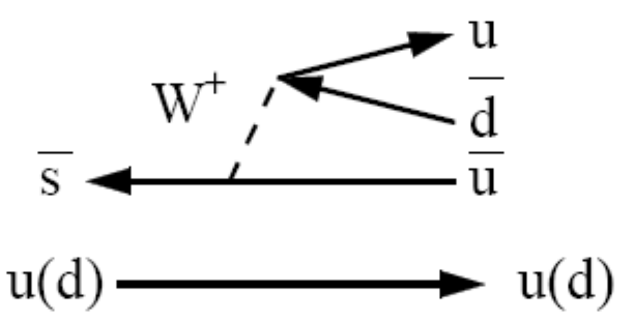


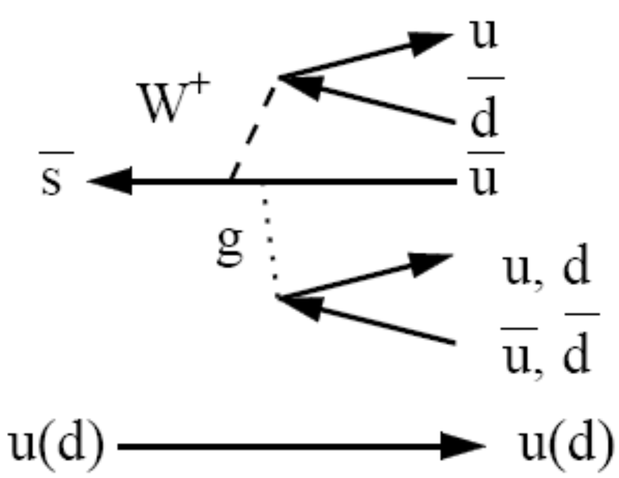
box diagrams



hadronic decays:

Tree diagrams



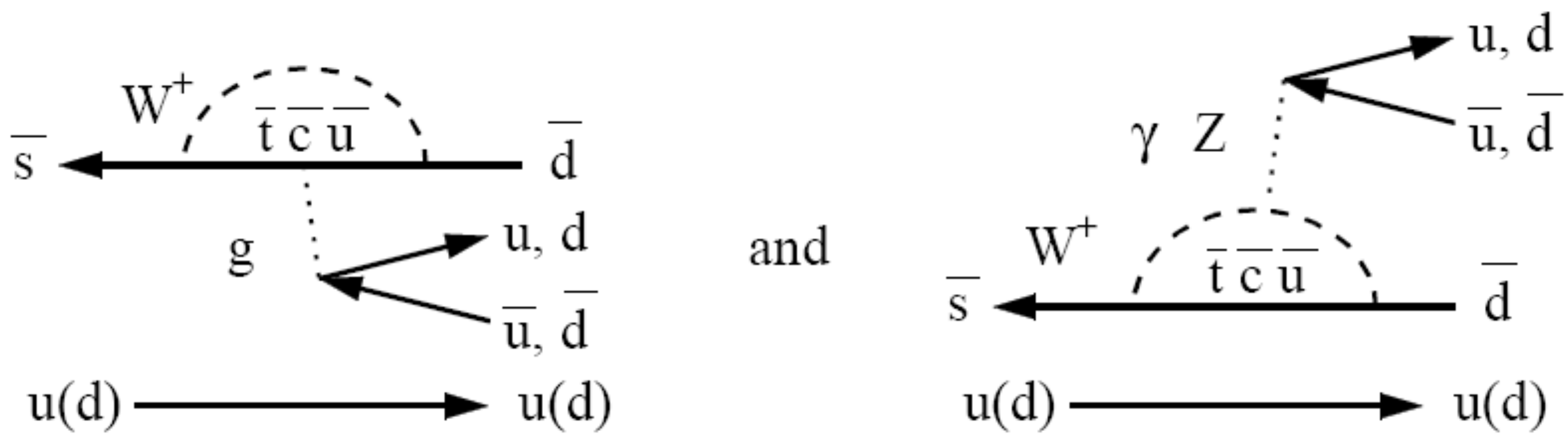


$K^+(K^0) \to \pi^+\pi^0\ (\pi^+\pi^-,\ \pi^0\pi^0)$

$K^+(K^0) \to \pi^+\pi^0\pi^0,\ \pi^+\pi^+\pi^-$
$(\pi^+\pi^-\pi^0,\ \pi^0\pi^0\pi^0)$

+ c.c.

Penguin diagrams

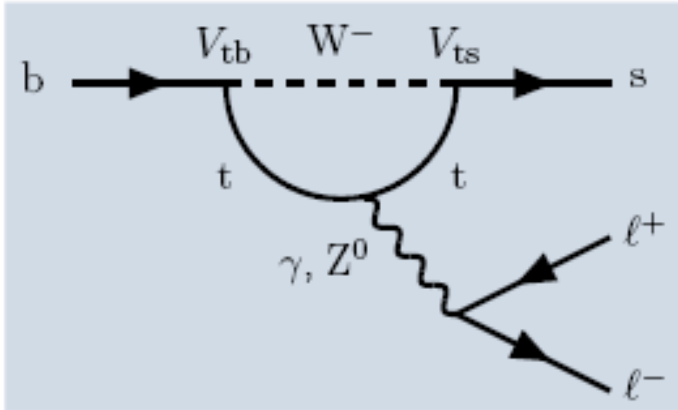


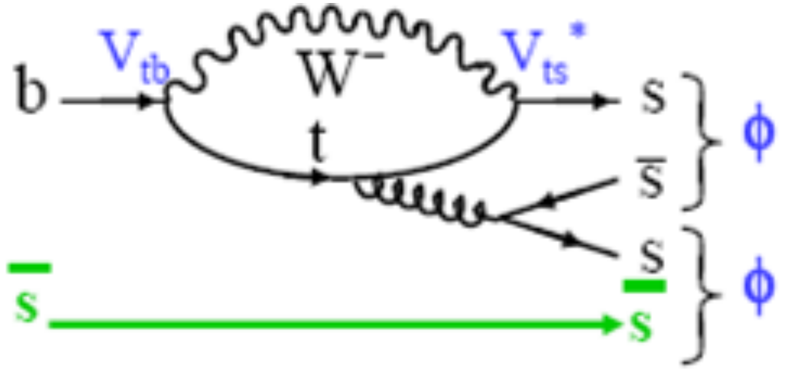
$$K^+(K^0) \to \pi^+\pi^0\,(\pi^+\pi^-,\, \pi^0\pi^0)$$

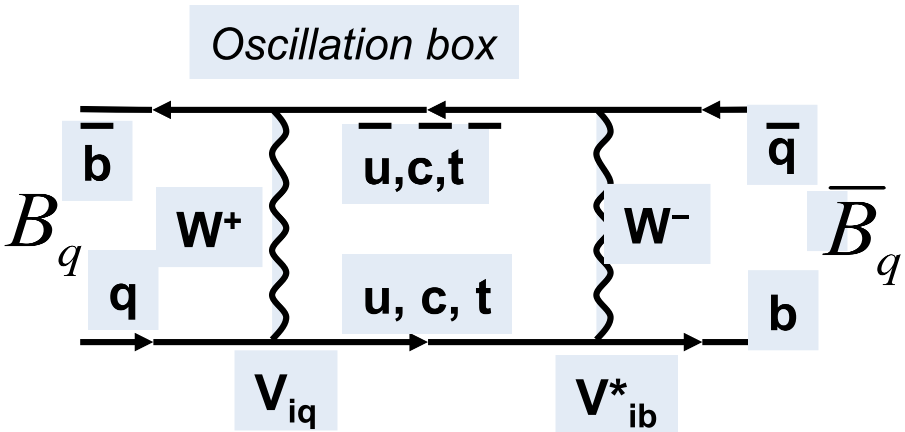
+ more gluons

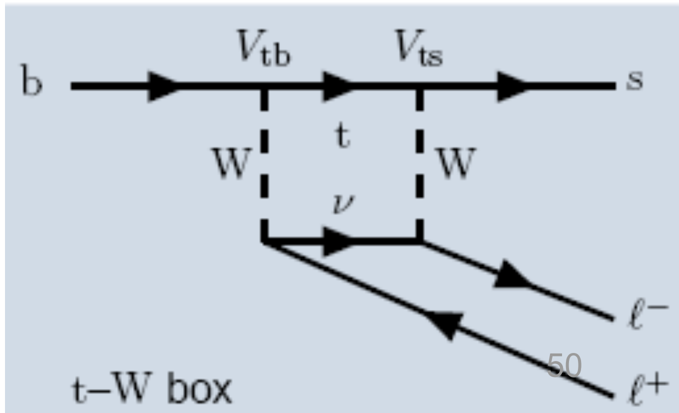
Examples of B decays mediated by loop diagrams

- *$B_{d,s}$ oscillations: box diagram*
- *Penguin diagrams:*
  - *Radiative penguin: $B_s \rightarrow \phi\gamma$*
  - *Electroweak penguin: $B \rightarrow K^*\mu\mu$*
  - *Strong penguin: $B \rightarrow \pi\pi$, $B_s \rightarrow \phi\phi$*









Physics of loops

- ***Loops can be explored in rare flavour decays***
- ***Why is it important to study loop processes in general?***

Loop processes contain *loop momentum integrals*
and hence can indirectly probe physics at *large mass scale*

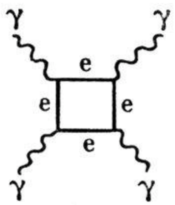
Example: quantum electrodynamics at small distances or in strong fields is sensitive to the electron mass in loops

a) the potential between static sources deviates from Coloumb law at small distances:

 + 

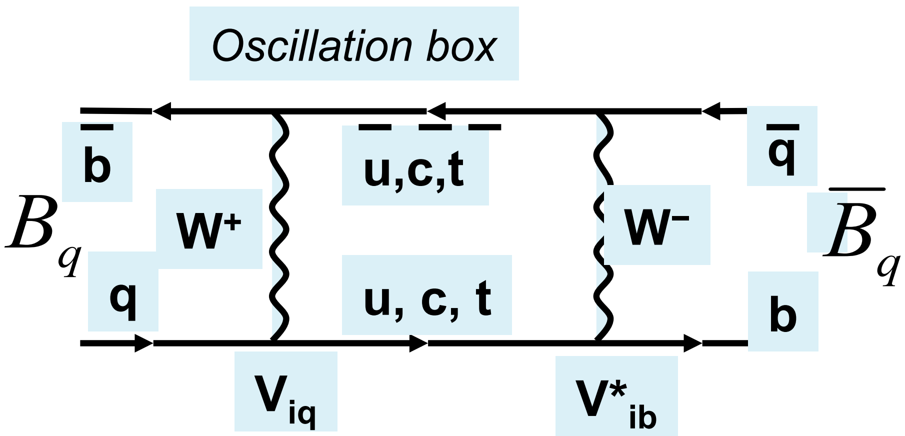
$$V(r) = \frac{\alpha}{r}\left[1 + \frac{2\alpha}{3\pi}\log\frac{1}{cm_e r}\right]$$

b) the energy stored by the static magnetic field is different from its classical value: $H^2/2$

$\varepsilon_{classic}$ + 

$$\varepsilon = \frac{H^2}{2}\left[1 - \frac{\alpha^2 H^2}{45\pi m_e^4}\right]$$

Analogously rare B-decays mediated by loop processes are sensitive to heavy particles masses and couplings: logarithmically for radiative penguins and power-like for box diagrams. However the concrete form of functional dependence is much more complicated than in considered simple examples.

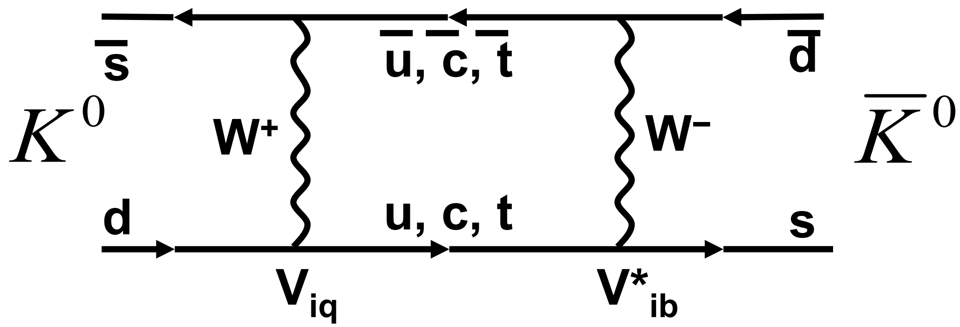


- *Diverging diagrams*
- *GIM cancellation*
- $\sim m_q^2$

Loop processes contain sums over all relevant degrees of freedom (Lorentz structure of the interaction, symmetries related to New particles etc…).

Example: neutral kaon oscillations

Neutral K-mesons made of d and anti-s quarks oscillate in vacuum with the frequency ~ 10^{10} sec^{-1} because of the following loop process, mediated by "box" diagram:



Notice that it is the same diagram which describes oscillations of B-mesons if we replace s-quark by b-quark!

Suppose we know nothing about the existence of heavy c- and t-quarks.

Then naïve estimate of the box diagram with one internal u-quark gives for the level splitting Δm_{12} (which is nothing but the oscillation frequency)

$$G_2 \sim \frac{\Delta m_{12}}{f_K^2 m_K} \sim G_F^2 \cdot M_W^2 \sim 10^{-6}\ GeV^{-2}$$

while experimental result is $G_2 \sim 10^{-13}\ GeV^{-2}$ It seems we have a problem…

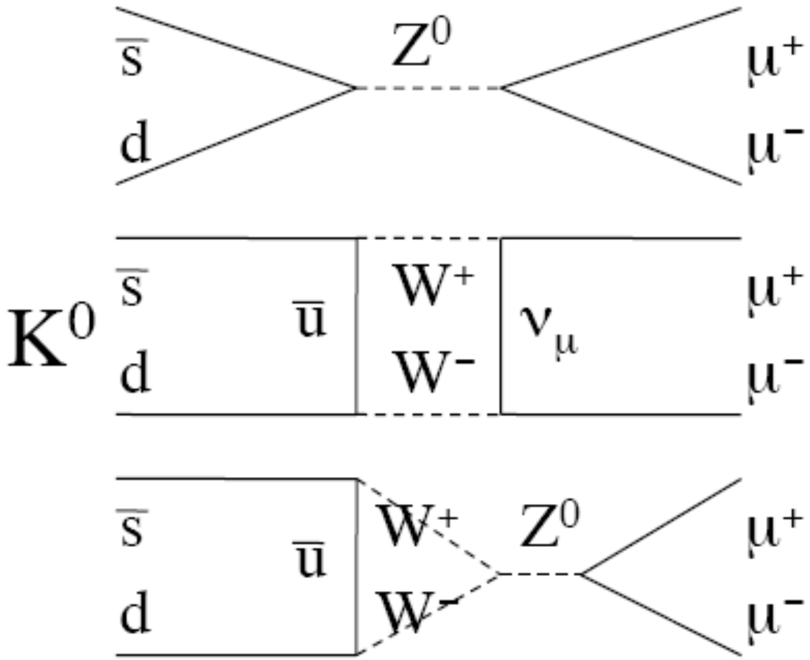
Solution: GIM - *S.Glashow, J.Iliopoulos, L.Maiani, 1970*
Box diagram with internal c-quark cancels the one with u-quark (up to the quarks mass difference):

$$G_2 \sim \frac{G_F^2}{16\pi^2}(m_c - m_u)^2 \sin^2\theta \cos^2\theta$$

Flavour Physics

Excellent track record to probe high energy scale

Very suppressed $K_L \rightarrow \mu^+\mu^-$

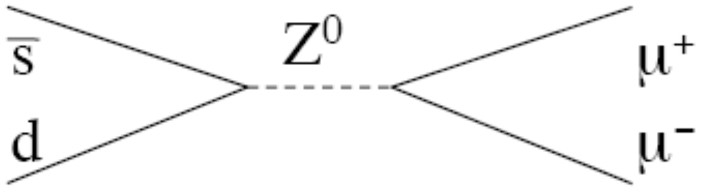


?

Flavour Physics

Excellent track record to probe high energy scale

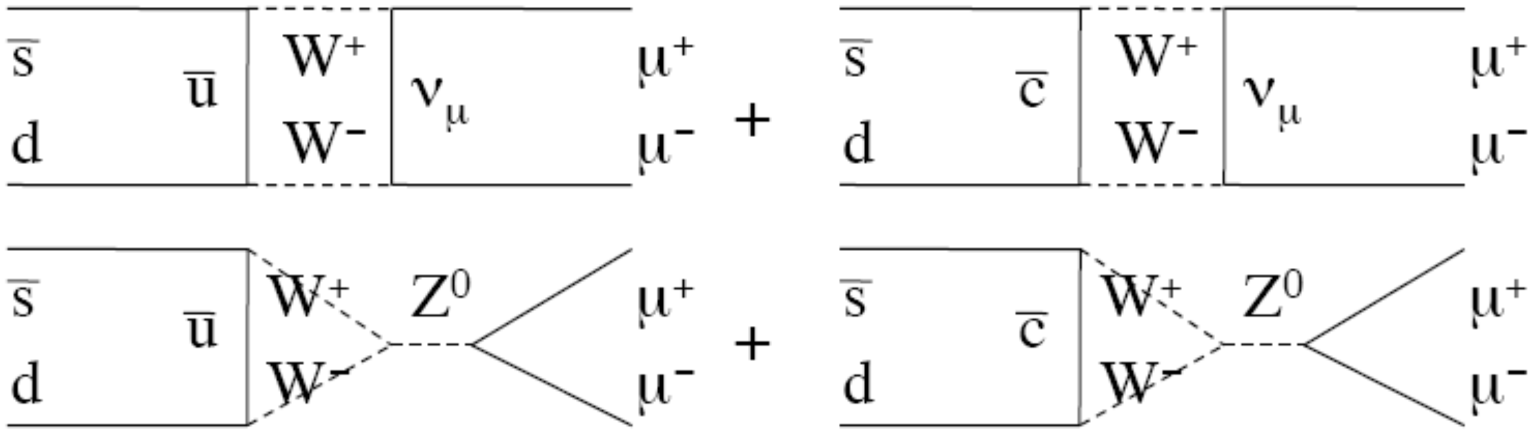
Very suppressed $K_L \rightarrow \mu^+\mu^-$ $\Rightarrow$ SU(2) doublet structure (GIM)



$= 0$

$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix}$

K^0



$= 0$ if $m_u = m_c$

Flavour Physics

Excellent track record to probe high energy scale

Very suppressed $K_L \rightarrow \mu^+\mu^-$ $\Rightarrow$ SU(2) doublet structure (GIM)

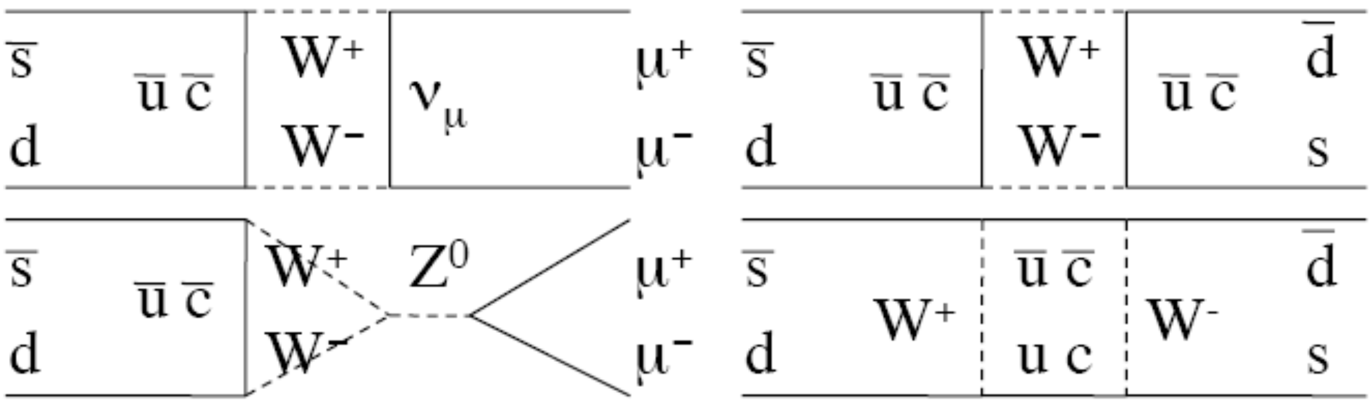
Δm_K and $Br(K_L \rightarrow \mu^+\mu^-)$

Flavour Physics

Excellent track record to probe high energy scale

Very suppressed $K_L \to \mu^+\mu^-$ $\Rightarrow$ SU(2) doublet structure (GIM)

Δm_K and $Br(K_L \to \mu^+\mu^-)$ $\Rightarrow$ charm mass ~1.5 GeV/c^2



$Br(K^0 \to \mu^+\mu^-) = F(m_c, \ldots)$ $\quad$ $\Delta m_K = G(m_c, \ldots)$

Flavour Physics

Excellent track record to probe high energy scale

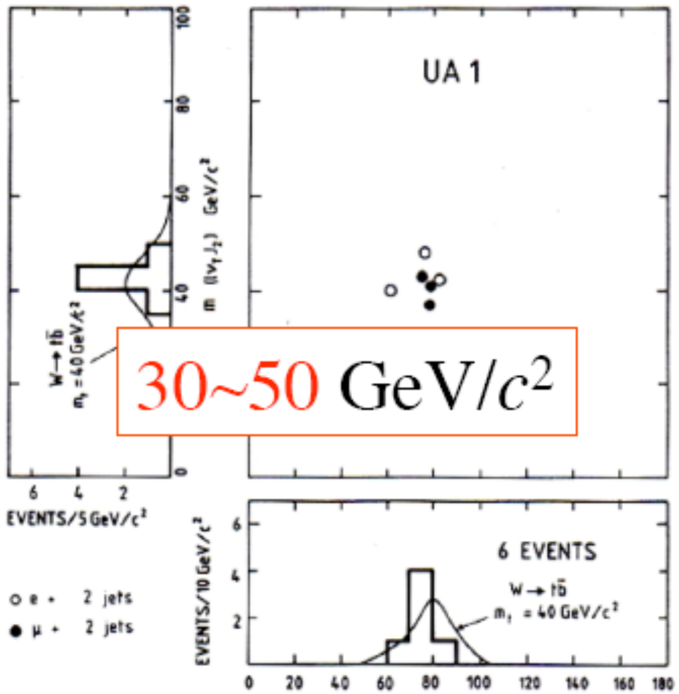
Very suppressed $K_L \to \mu^+\mu^-$	$\Rightarrow$ SU(2) doublet structure (GIM)
Δm_K and $Br(K_L \to \mu^+\mu^-)$	$\Rightarrow$ charm mass
CPV and very suppressed $B \to \mu^+\mu^-$	$\Rightarrow$ third family
Δm_B	$\Rightarrow$ top mass

before observing directly c, b or t

History of m_t

UA1, 1984

$$W^+ \to t\,\bar{b}, \quad \bar{b} \to \text{jet}, \quad t \to b l \nu, \quad b \to \text{jet}$$



30~50 GeV/c^2

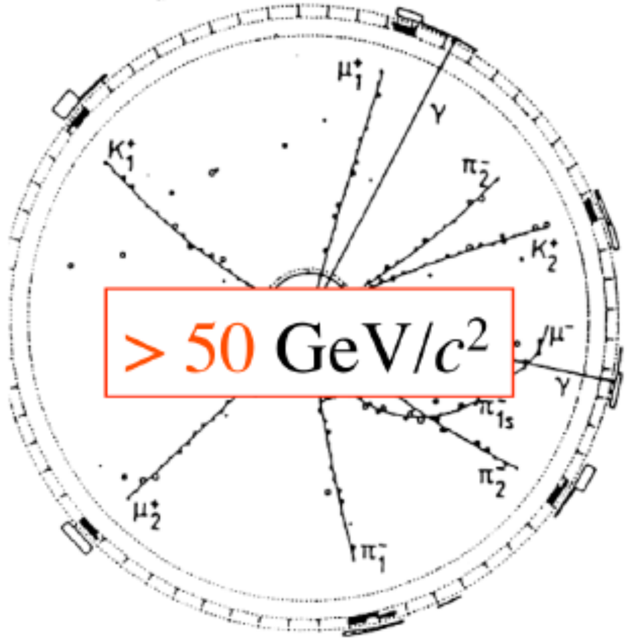
Phys. Lett. 147B (1984) 493

ARGUS, 1987

$$\Upsilon(4S) \to B_d^0 \overline{B}_d^0$$
$$\to B_d^0 B_d^0 \text{ or } \overline{B}_d^0 \overline{B}_d^0$$
$$\to \ell^+\ell^+ \text{ or } \ell^-\ell^-$$

24.8±7.6±3.8

$\Delta m(B_d) \sim 100 \times \Delta m(K^0)$



> 50 GeV/c^2

Phys. Lett. B 192 (1987) 245

LEP
electroweak fit
150~210 GeV/c^2

1995
CDF
175±8±10 GeV/c^2
D0
199^{+19}_{-21}±22 GeV/c^2

Comparison of calculated G_2 with experimentally measured Δm_{12} leads to correct prediction for $m_c \sim 1\,GeV$

This is how it actually happened: GIM mechanism was suggested in 1970, while direct experimental discovery of c-quark took place only in 1974!

Historical remark #1. Perhaps even more spectacular is that the famous Kobayashi-Maskawa paper where the quarks of third generation (b- and t-quarks) and current paradigm of CP-violation were introduced was also published a few months before c-quark discovery (and about four years before b-quark discovery).

Historical remark #2. Original idea about possible fourth quark (c-quark) Was suggested by M.Gell-Mann in his original '1964 paper devoted to the quark model with three light quarks (u-, d-, and s- quarks) on aesthetic grounds of symmetry between quarks and leptons.

Historical remark #3. The analogous mixing matrix in lepton sector was proposed by Z.Maki, M.Nakagawa and S.Sakata in 1962, i.e. well before CKM!