Flame acceleration by turbulence in SNIa

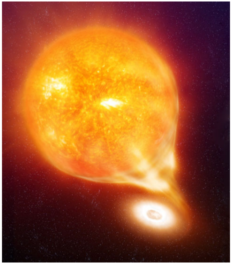
Glazyrin S.I.   Blinnikov S.I.

Institute for Theoretical and Experimental Physics
glazyrin@itep.ru

Heavy elements nucleosynthesis and galactic chemical evolution
(Moscow, September 9–10, 2013)

Supernovae Ia (SNIa)

- Several scenario exist. Basic: semi–degenerate (SD), double–degenerate (DD). Here the only SD variant is considered
- A thermonuclear explosion of a white dwarf with mass $\sim M_{\mathrm{Ch}}$
- The general physical problem: describe burning of a white dwarf



The problem of the flame propagation

- The flame is ignited near the centre of a WD
- For successful explosion and reproducing observables the transition to detonation (DDT) is required
- The flame is subject to several instabilities, thought to be responsible for the DDT
- As a first step the flame should be accelerated to $\sim 30\%$ of the speed of sound $\Longrightarrow$ acceleration ~ 30 times

Stationary burning waves

Deflagration wave (a flame)

- Heat spreads by some diffusion-like process (thermocondictivity, diffusion etc.) and burns up nearby layers
- velocity $\ll c_S$

Detonation wave

- A shock wave increases temperature (pressure, density etc.) and medium starts to burn
- velocity $> c_S$

Instabilities of the flame

Spatial scale: δ_{flame} → R_{WD}

- Pulsational (1D)

  Bychkov, Liberman ApJ (1995)

  Glazyrin, Blinnikov, Dolgov MNRAS (2013)

- Landau–Darries (2D, 3D)

  Röpke et al. A&A (2004), Bell et al. ApJ (2004)
  Glazyrin, Blinnikov HEA-2011

  Glazyrin Astron. Lett. (2013)

- Rayleigh–Taylor–Landau (2D, 3D)

  Bell et al. ApJ (2004)

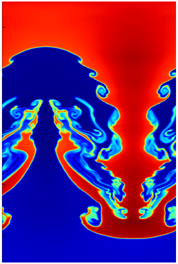
  Zingale et al. ApJ (2005)

- Interaction with the turbulence
  Röpke ApJ (2007)

  Glazyrin arXiv:1307.8357

Rayleigh–Taylor–Landau instability

Rayleigh (1883), Taylor (1950), Landau (1944)



$\mathrm{v}_n = 0$

- Occurrence conditions

$$\nabla\rho \cdot g < 0$$

$$g > g_{\mathrm{cr}}$$

- Linear stage

$$\omega = k u_n \frac{\mu}{1+\mu}\left(\sqrt{1+\mu-\frac{1}{\mu}-\frac{\mu^2-1}{\mu^2}\frac{g}{k u_n^2}}-1\right)$$

- Non-linear stage

$$\text{for} \quad \mathrm{v}_n = 0: \quad \Delta z = \alpha A_t g t^2$$

Turbulence in SNIa

Scales: $R_{\mathrm{star}} \sim 1000$ km, $\delta \sim 10^{-4}$ cm, $\mathrm{Re} \sim 10^{14}$

$\Downarrow$

DNS is not possible

Approaches:

- ILES
- LES (Niemeyer, Hillebrandt ApJ (1995), Schmidt et al. A&A (2006))
- Linear Eddy Model (Woosley et al. ApJ (2009))
- Turbulence models: k-ϵ

How turbulence influence burning?

The Gibson scale

$$\mathrm{v}'(l_G) = u_n$$

The Karlovitz number

$$\mathrm{Ka} = \left(\frac{\delta}{l_G}\right)^{1/2}$$

- Flamelet regime

$$\mathrm{Ka} \lesssim 1$$

- Well–stirred reactor regime

$$\mathrm{Ka} \gg 1, \quad \mathrm{Da} = \frac{L}{u_n \tau_{\mathrm{nuc}}} < 1$$

- Stirred flame regime

$$\mathrm{Ka} \gg 1, \quad \mathrm{Da} > 1$$

How turbulence influence burning?

- Laminar flame

$$\tau_{\rm nucl} = \tau_{\rm cond} \quad \rightarrow \quad u_n = \sqrt{\frac{\kappa \dot{S}}{\rho q}}$$

- Flamelet regime: the flame is curved

$$\mathrm{v}_{\rm turb} = u_n \frac{A}{A_0}$$

- Well–stirred reactor regime

burning in the whole region

- Stirred flame regime

$$\mathrm{v}_{\rm turb} = \sqrt{\frac{(\kappa + \kappa_{\rm turb})\dot{S}}{\rho q}}$$

Flamelet regime: Yakhot's formula

Yakhot Comb. Sci. Tech. (1988)
The equation of flame evolution

$$\partial_t G + \mathbf{v}\nabla G = -u_0|\nabla G|,$$

$\mathbf{v}$ is velocity of a developed turbulence. With the renorm-group analysis he obtained a simple result

$$\frac{\mathrm{v}_t}{u_n} = \exp\left(\frac{\overline{\mathrm{v}'^2}}{\mathrm{v}_t^2}\right)$$

Limiting cases:

$$\overline{u'^2} \to 0: \quad \mathrm{v}_t = u_n\left(1 + \frac{\overline{\mathrm{v}'^2}}{u_n^2}\right) \qquad \overline{u'^2} \to \infty: \quad \mathrm{v}_t = \sqrt{\overline{\mathrm{v}'^2}}$$

The procedure to derive a model of turbulence

$$\partial_t \rho + \partial_i(\rho \mathrm{v}_i) = 0,$$

$$\partial_t(\rho \mathrm{v}_i) + \partial_j(\rho \mathrm{v}_i \mathrm{v}_j) + \partial_i p = \partial_j \tau_{ij},$$

$$\partial_t(\rho e) + \partial_i\left(\rho e \mathrm{v}_i\right) + p\partial_i \mathrm{v}_i + \underline{\partial_i Q_i} = \tau_{ij}\partial_j \mathrm{v}_i + \underline{\dot{S}}.$$

Averaging rules:

$$\overline{A}(\mathbf{x}, t) = \frac{1}{T}\int\limits_{-T/2}^{T/2} A(\mathbf{x}, t+\tau)d\tau,$$

$$A \equiv \overline{A} + A', \quad A \equiv \tilde{A} + A'', \quad \tilde{A} \equiv \frac{\overline{\rho A}}{\overline{\rho}}$$

k-ϵ model

New dynamical quantities

$$\overline{\rho} k \equiv \frac{\overline{\rho \mathrm{v}''^2}}{2}, \quad \overline{\rho}\epsilon \equiv \overline{\tau'_{ij}\partial_j \mathrm{v}'_i}$$

then turbulent viscosity

$$D = c_D \frac{k^2}{\epsilon}$$

"Gradient approximation" (Belenkii, Fradkin Trudi FIAN(1965)):

$$\overline{\mathrm{v}''_i A'} \sim -D\partial_i A$$

As an example, Reynolds tensor expands as

$$R_{ij} = \overline{\rho \mathrm{v}''_i \mathrm{v}''_j} = -\rho D \left(\partial_i \mathrm{v}_j + \partial_j \mathrm{v}_i - \frac{2}{3}\delta_{ij}\partial_k \mathrm{v}_k \right) + \frac{2}{3}\rho k \delta_{ij}$$

k-ϵ model

$$\partial_t \rho + \partial_i(\rho \mathrm{v}_i) = 0,$$
$$\partial_t(\rho \mathrm{v}_i) + \partial_j\left(\rho \mathrm{v}_i \mathrm{v}_j + p\delta_{ij}\right) = -\partial_j R_{ij},$$
$$\partial_t E + \partial_i(\mathrm{v}_i(E+p)) = -G_2 + \rho\epsilon + \partial_i(p a_i - Q_i^T),$$

$$\partial_t(\rho k) + \partial_i(\rho k \mathrm{v}_i) = G_1 + G_2 - \rho\epsilon + \partial_i(\rho c_k D \partial_i k),$$
$$\partial_t(\rho\epsilon) + \partial_i(\rho\epsilon \mathrm{v}_i) = \frac{\epsilon}{k}\left(c_{\epsilon 1} G_1 + c_{\epsilon 2} G_2 - c_{\epsilon 3}\rho\epsilon\right) + \partial_i(\rho c_\epsilon D \partial_i \epsilon),$$

$$R_{ij} = -\rho D\left(\partial_i \mathrm{v}_j + \partial_j \mathrm{v}_i - \frac{2}{3}\delta_{ij}\partial_k \mathrm{v}_k\right) + \frac{2}{3}\rho k \delta_{ij},$$

$$E = \rho e + \frac{\rho \mathrm{v}^2}{2}, \quad D = c_D \frac{k^2}{\epsilon}, \quad a_i = -c_\alpha D \frac{\partial_i \rho}{\rho},$$

$$G_1 = -R_{ij}\partial_i \mathrm{v}_j, \quad G_2 = a_i \partial_i p, \quad Q_i^T = -c_e \rho D \partial_i e.$$

k-ϵ model: constants

Guzhova et al. VANT TPF (2005)

The model constants were fitted to match a Rayleigh–Taylor and a Kelvin–Helmholtz mixing processes in the best way:

$$c_\alpha = 1.7 \quad c_D = 0.12 \quad c_e = 3 \quad c_{\epsilon 1} = 1.15$$

$$c_{\epsilon 2} = 1 \quad c_{\epsilon 3} = 1.7 \quad c_k = c_\epsilon = 4/3$$

The flame description

$$\frac{dm_{\text{flame}}}{dt} = 4\pi r_{\text{fl}}^2 \rho \mathrm{v}_{\text{fl}}$$

$$\Delta Q = q\Delta m \quad \rightarrow \quad \partial_t(\rho e) = \ldots + \dot{S}$$

Flame velocity:

1. Normal velocity u_n from Timmes, Woosley ApJ (1992)
2. Turbulence contribution with Yakhot's formula

$$\frac{\mathrm{v}_{\text{fl}}}{u_n} = \exp\left(\frac{2k}{\mathrm{v}_{\text{fl}}^2}\right)$$

FRONT3D–Lagr1D

FRONT3D code — a 3D parallel open-source hydrocode
http://dau.itep.ru/sn/front3d

- Lagrangian 1D hydrodynamics (as a module)
- k-ϵ model of turbulence
- Flame

The problem setup

Initial profiles

1. $m_i,\ \rho_c,\ T(\rho) \to T_0 = \text{const}$
2. Integrating

$$4\pi r_i^2 \frac{p_{i+1/2} - p_{i-1/2}}{\Delta m_i} = -\frac{G m_i}{r_i^2},$$

$$\rho_{i+1/2} = \text{EOS}(p_{i+1/2}, T_{i+1/2}),$$

$$r_{i+1}^3 = r_i^3 + \frac{3}{4\pi}\frac{\Delta m_{i+1/2}}{\rho_{i+1/2}}.$$

Ignition & Initial turbulence

Nonaka et al. ApJ (2012):

$$r_{\text{ign}} = 50 \text{ km}$$
$$v_0'' = \sqrt{2k_0} = 16 \text{ km/s}$$
$$L = k_0^{3/2}/\epsilon_0 = 200 \text{ km}$$

Parameters

Potekhin, Chabrier A&A (2012):

$\rho_c = 2 \times 10^9$ g/cm^3

^{12}C:	$T_0 = 2.7 \times 10^8$ K
^{12}C+^{16}O:	$T_0 = 3.8 \times 10^8$ K

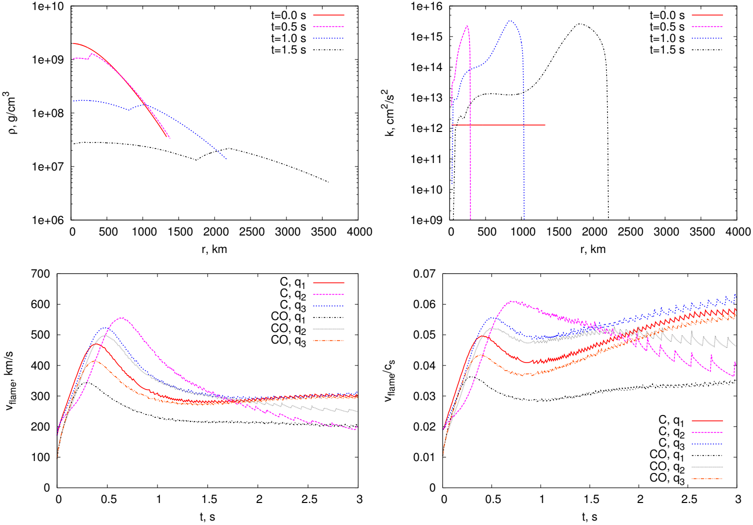
EOS: "Helmholtz tabular EOS" (Timmes, Swesty ApJS (2000))

Flame caloricities

C→Mg	$q_1 = 5.6 \times 10^{17}$ erg/g
C→Ni	$q_2 = 9.2 \times 10^{17}$ erg/g
NSE	$q_3 = 7.0 \times 10^{17}$ erg/g

Results

^{12}C, q_2:



Conclusions

- The k-ϵ model for turbulent flames in SNIa is presented
- The Rayleigh–Taylor–Landau instability generates turbulence with energy $\sim 10^{15}$ erg/g
- The turbulence accelerates flame (and maintains its velocity) up to ~ 300 km/s
- The laminar flame velocity and the laminar–turbulent relation have a little effect on the flame propagation

Thank you!