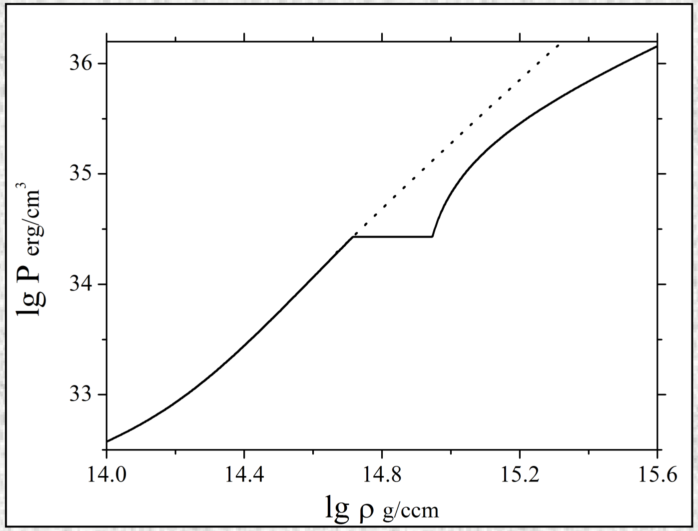
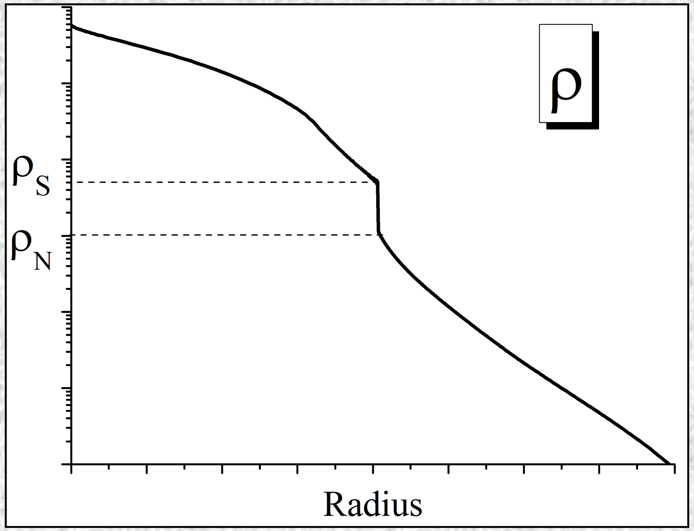
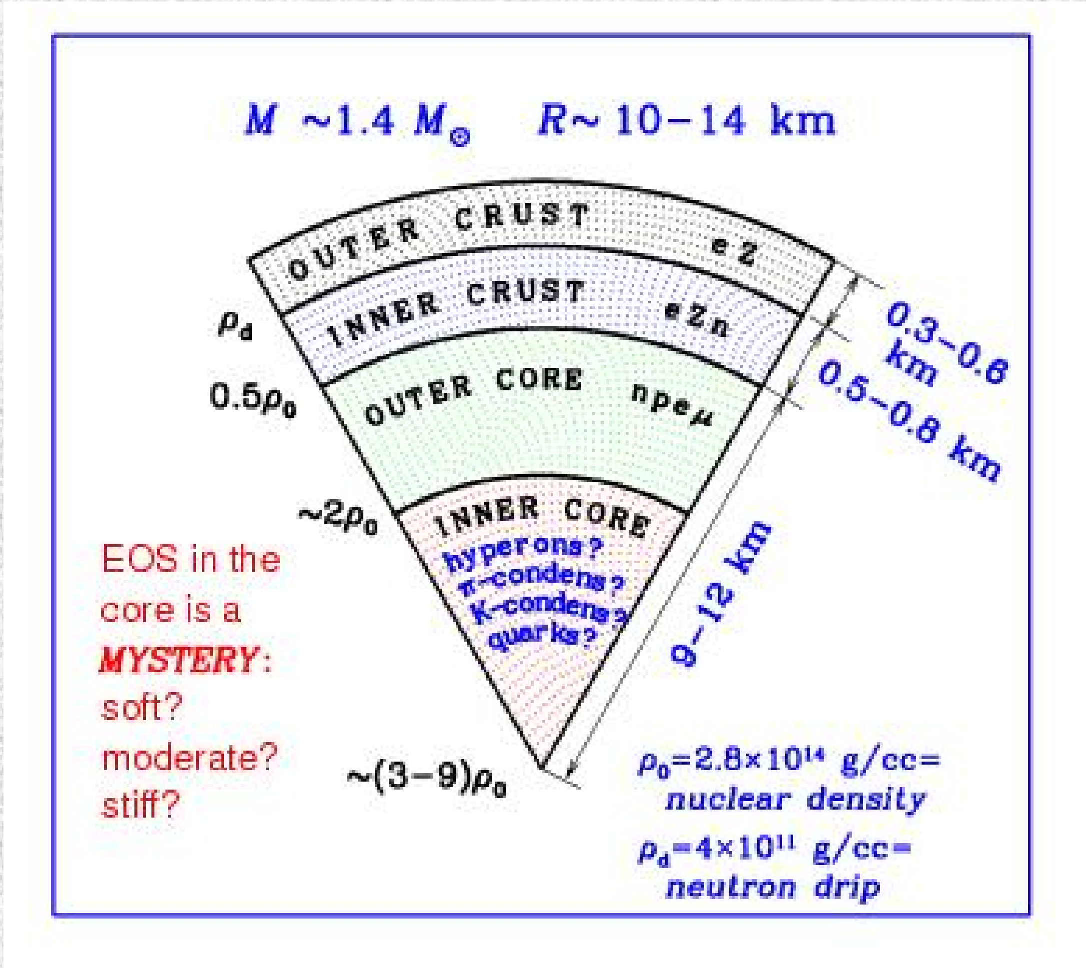
On an amazing feature of the mass-radius relation for superdense hybrid stars

A.V. Yudin, T.L. Razinkova, D.K. Nadyozhin, A.D. Dolgov

Possible phase transitions in matter at high density







Mass-radius diagram for hybrid stars

Phase transition:

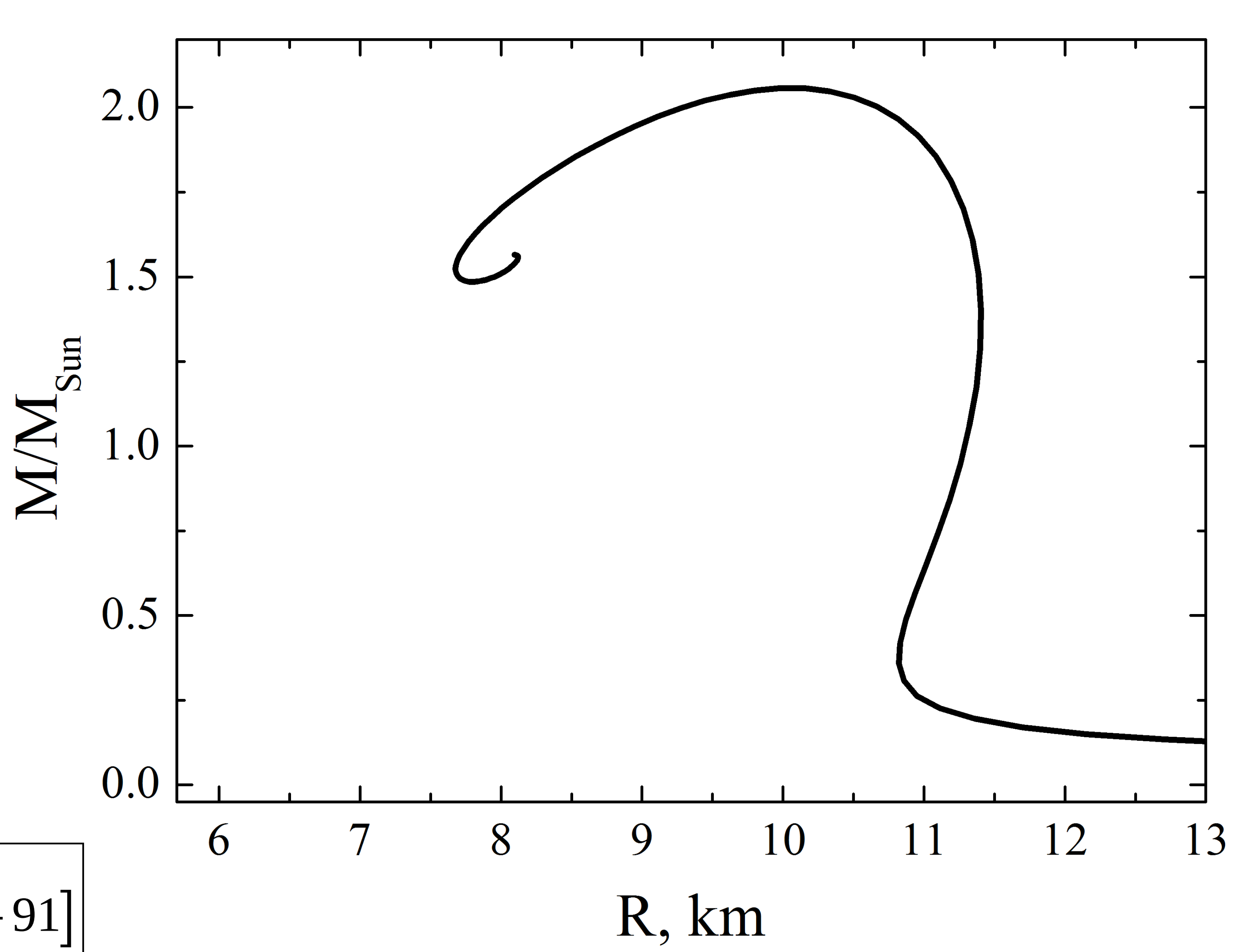
Nuclear Matter

↕

Quark Matter

$$P_q = \frac{1}{3}(E - 4B)$$

$$\frac{\rho_{pt}}{\rho_n} \approx -3 + \ln[B - 91]$$



Values of B are in MeV/fm^3

EOS for pure nuclear matter is from Douchin & Haensel, Astron. Astrophys., 380 151-167 (2001)

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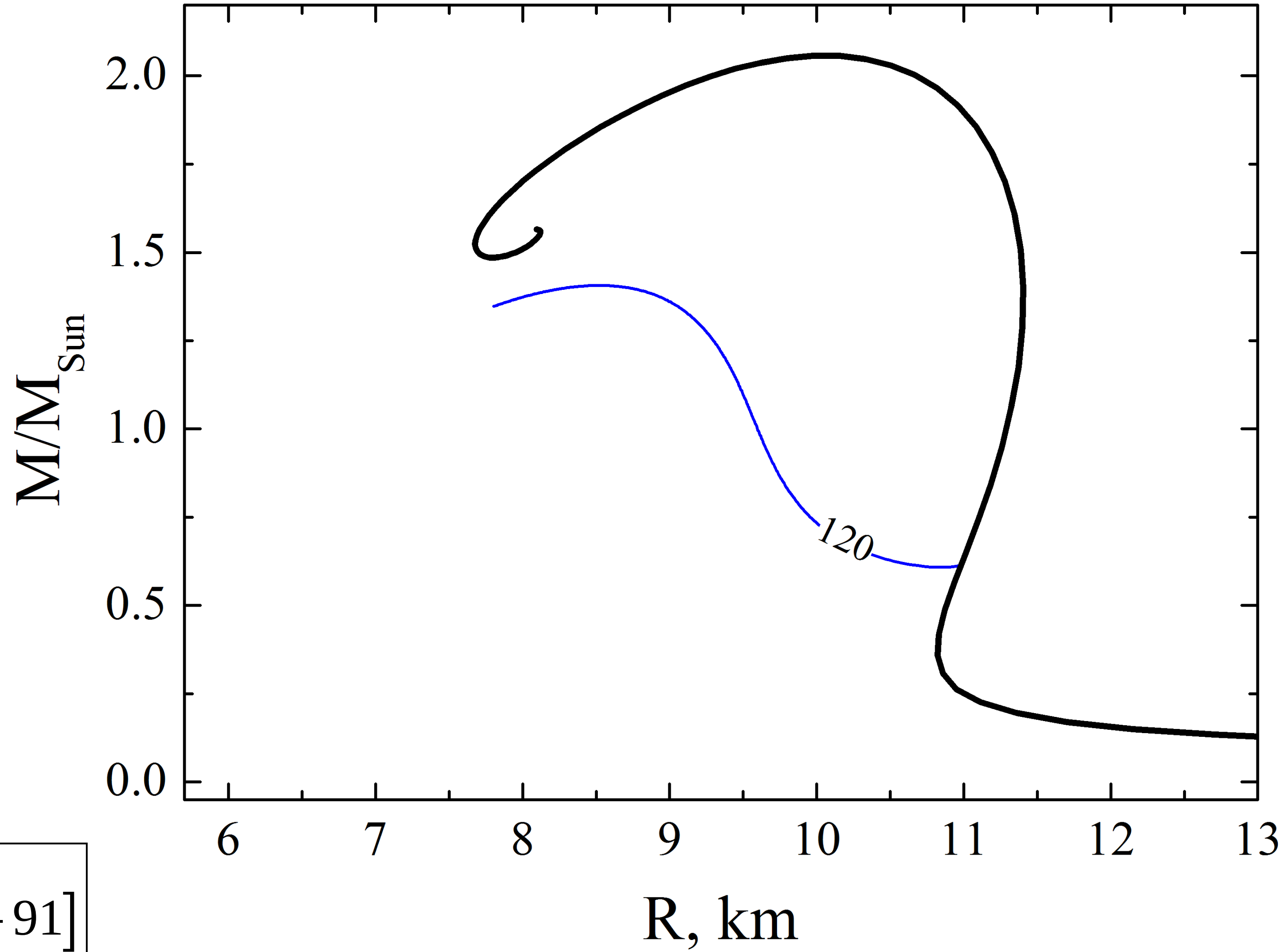

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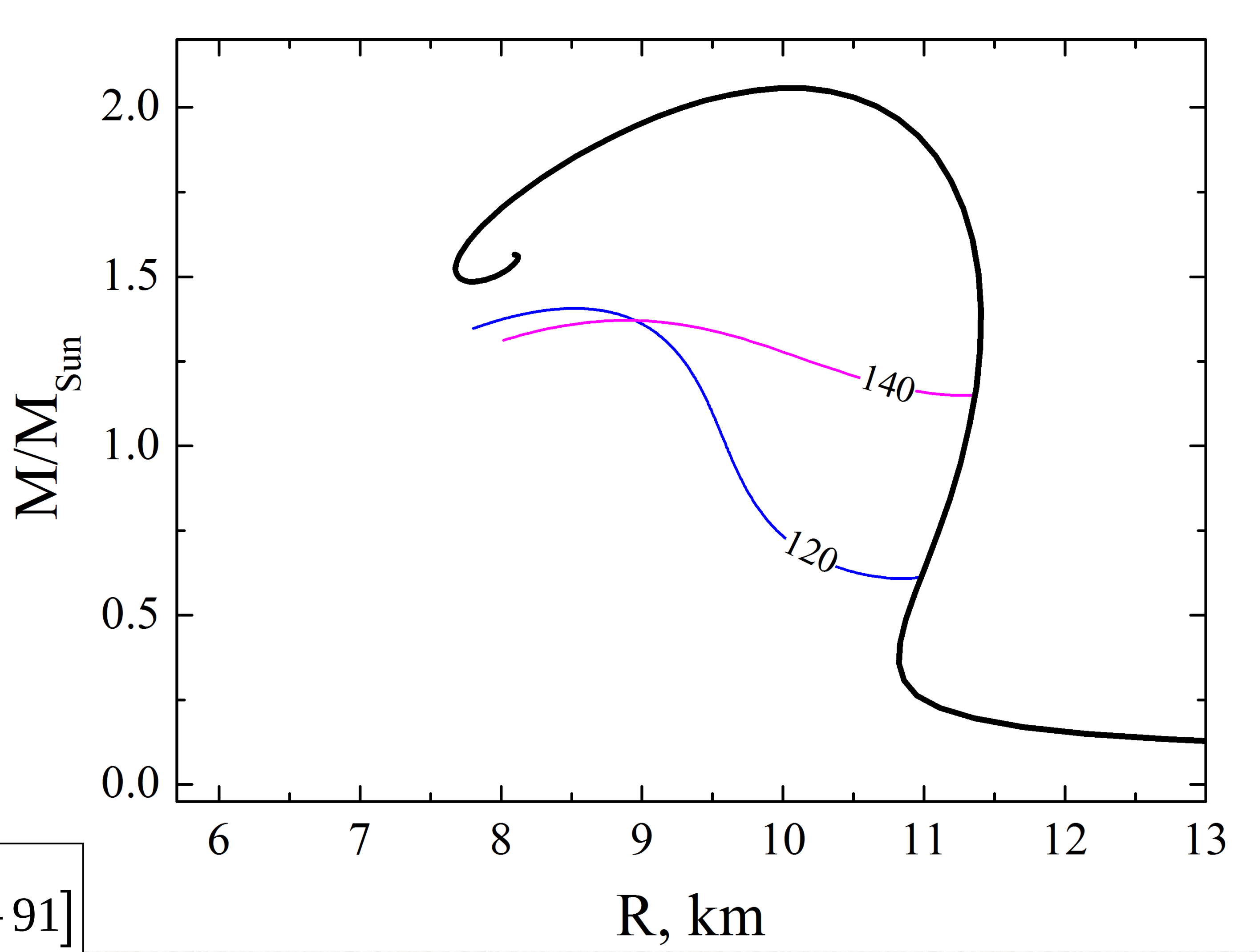

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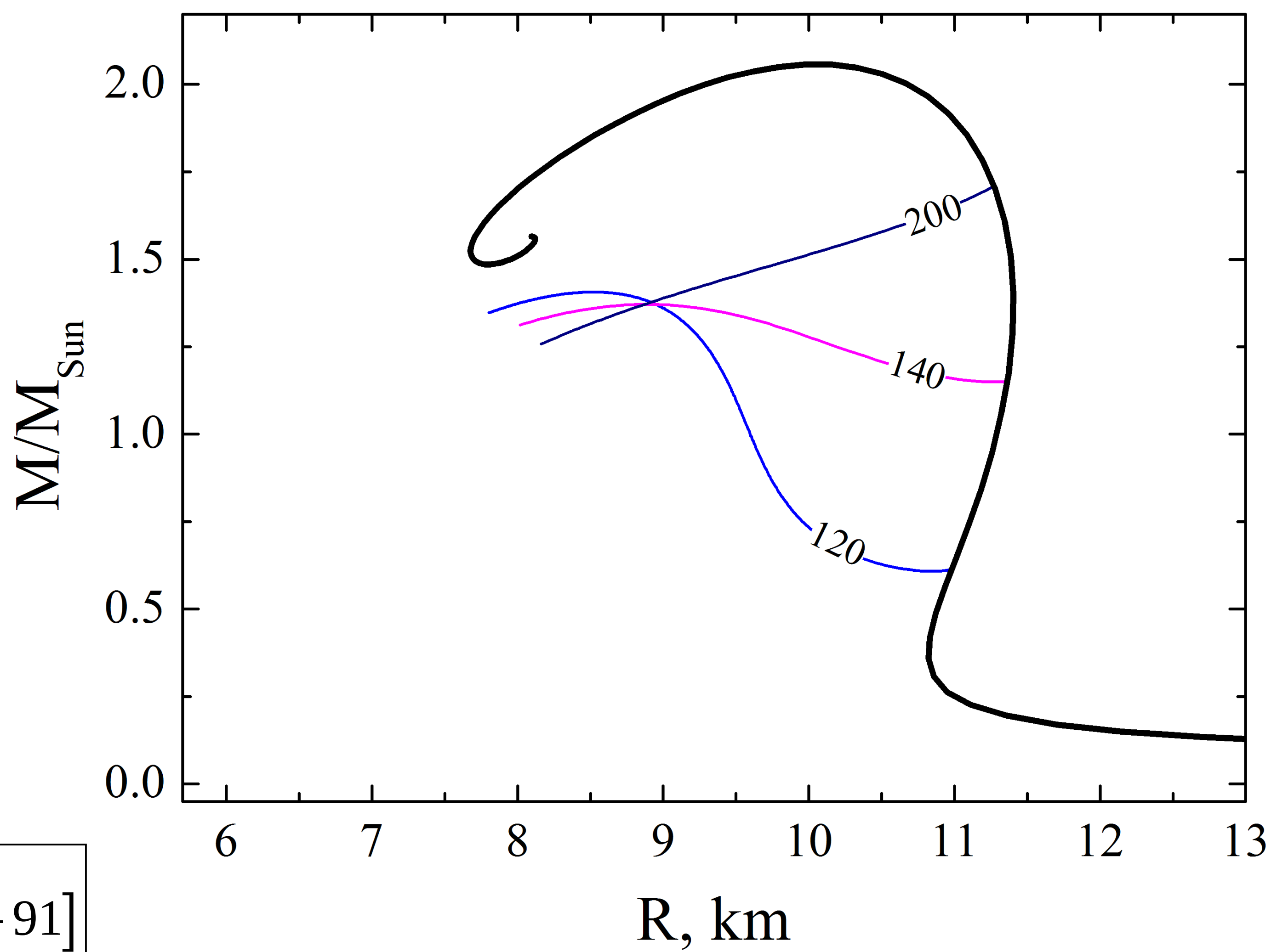

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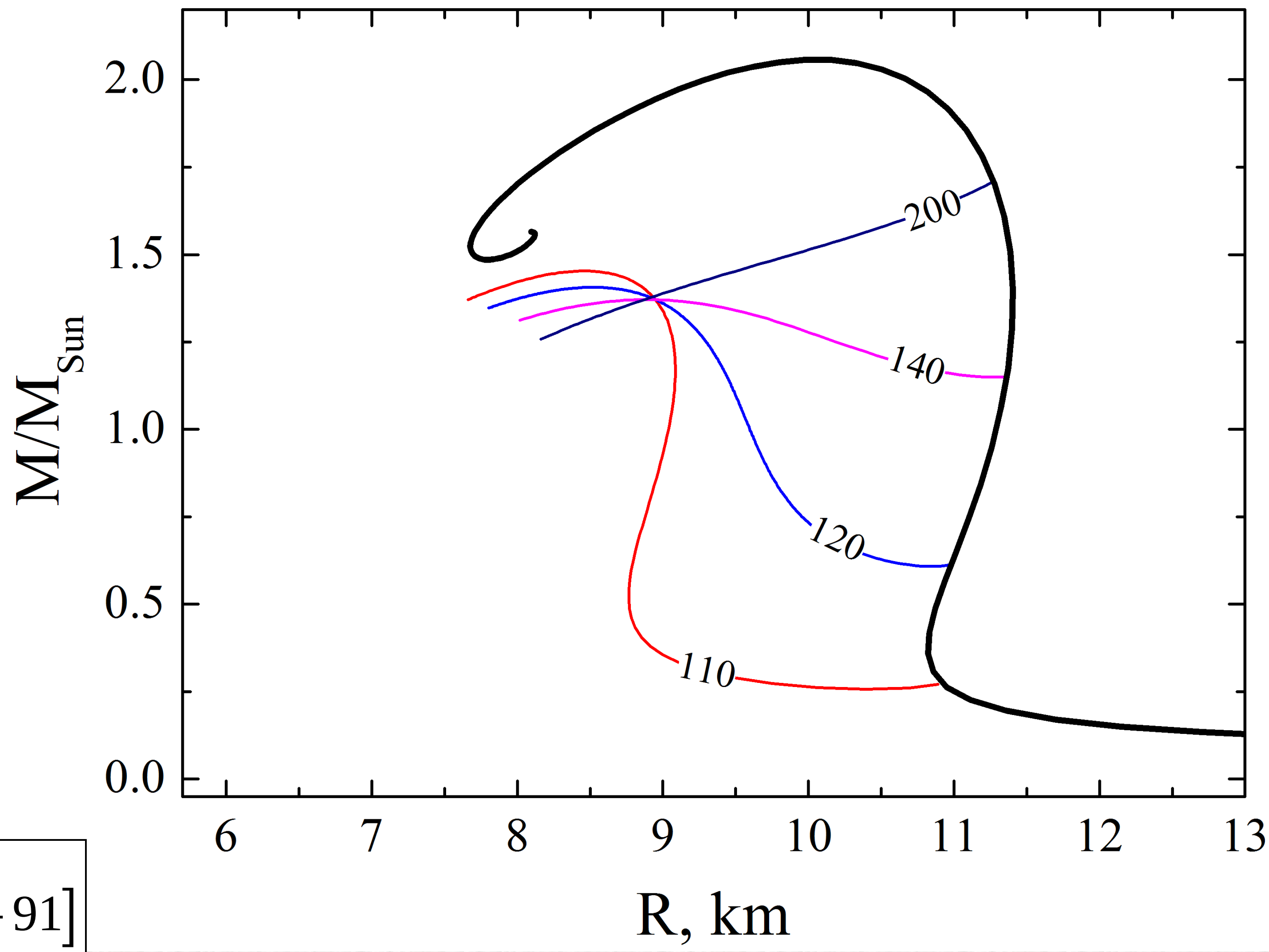

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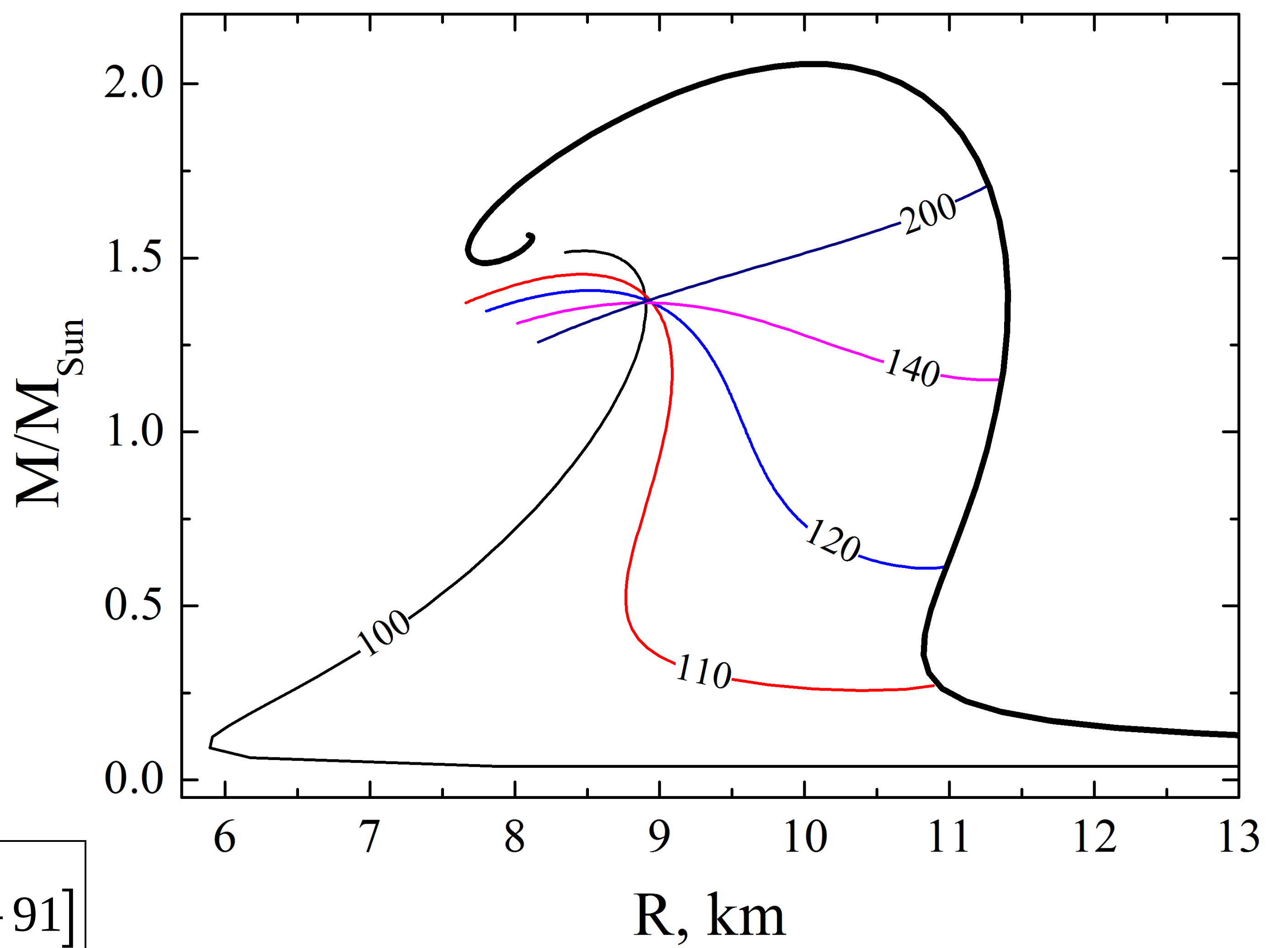

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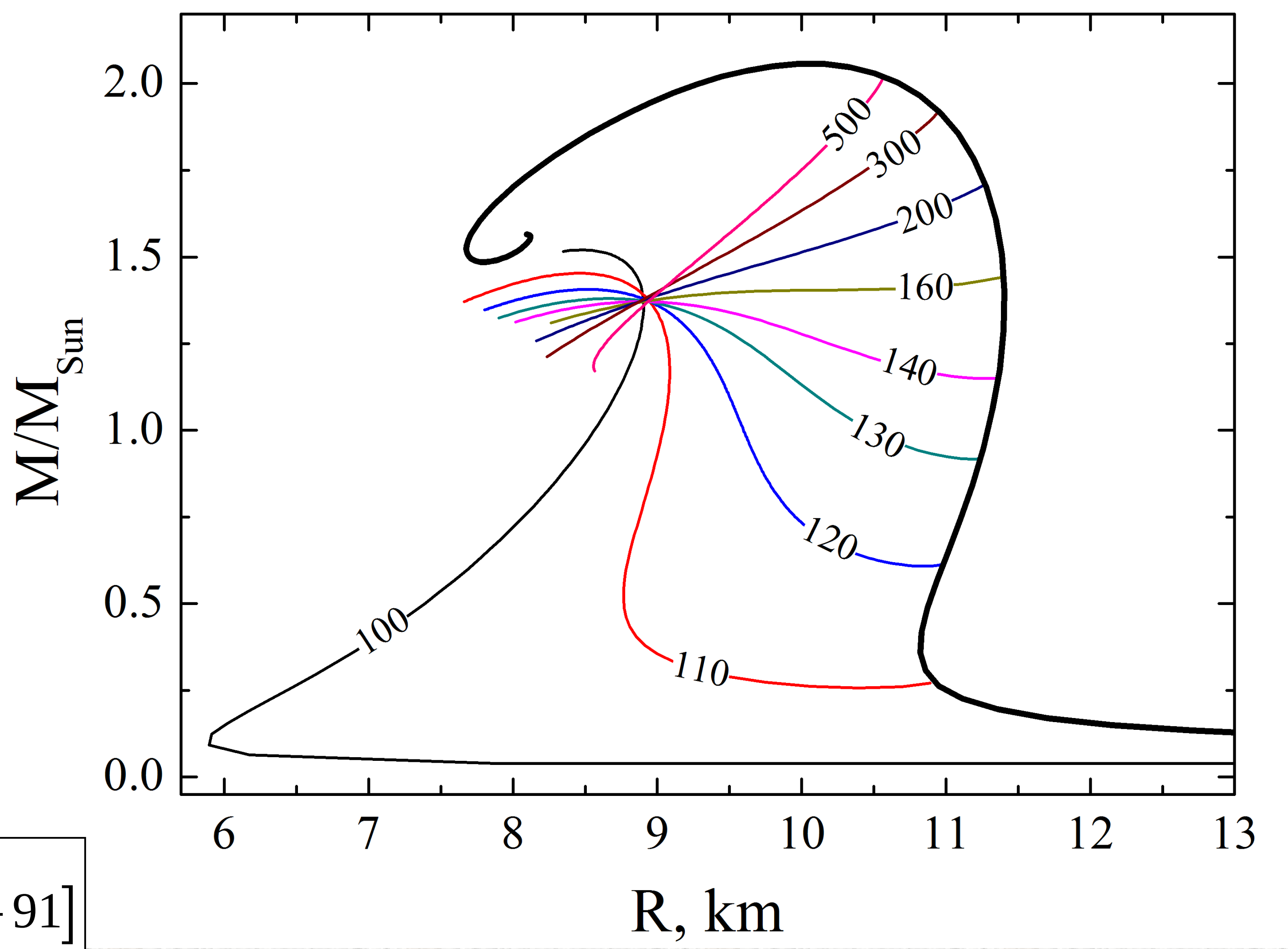

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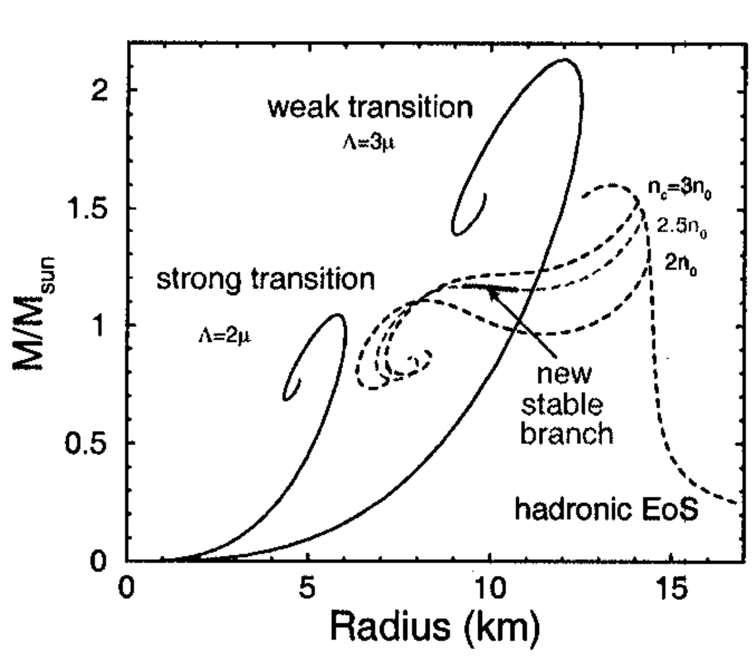
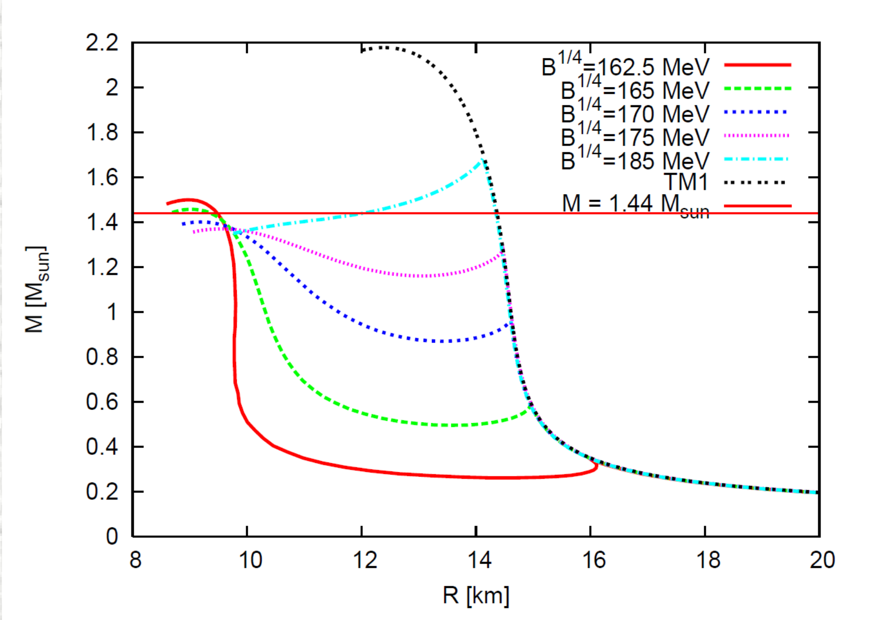
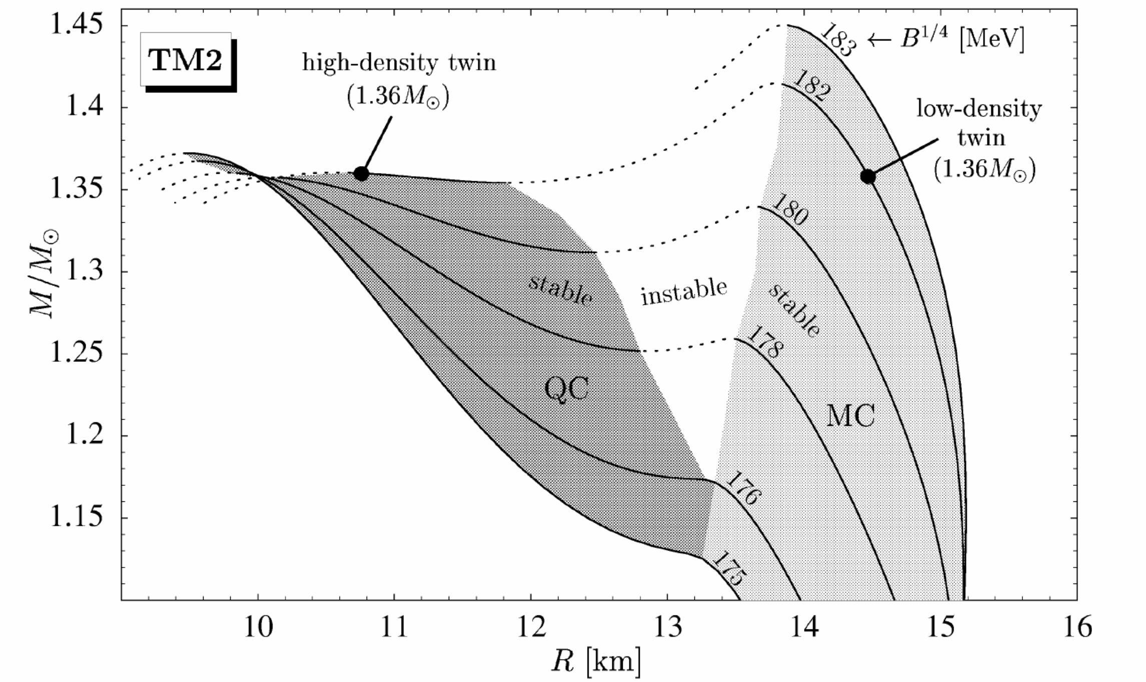
Fraga et al. Nucl. Phys. A, 702 (2002)



Sagert et al. J Phys G. 36, 6 (2009)

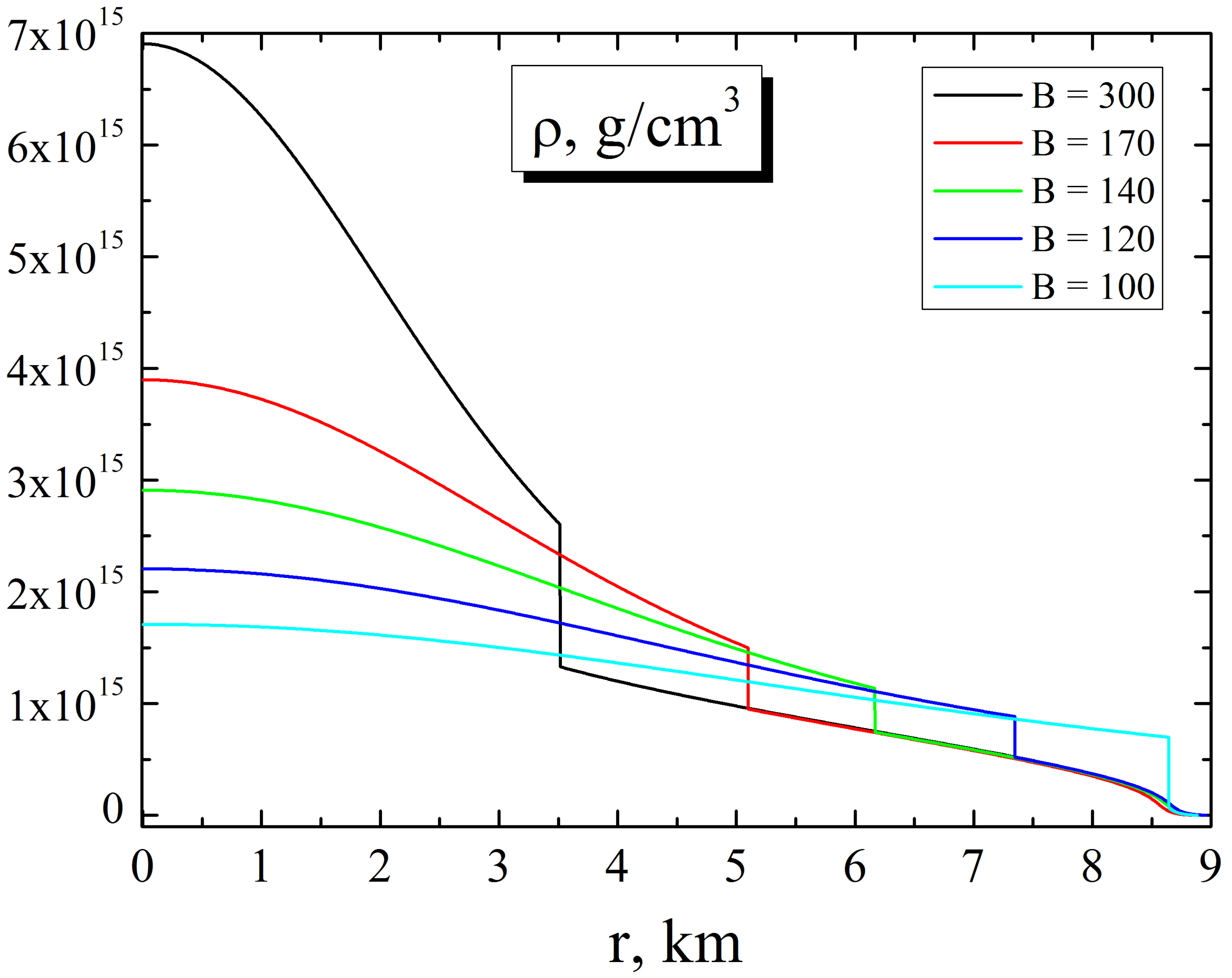


K. Schertler et al. / Nuclear Physics A 677 (2000) 463–490 483



The are other examples of such M-R diagram in the literature!

What is going on inside the star at the Magic Point?



OVT-equations of equilibrium

$$\begin{cases} \dfrac{dP}{dr} = -\dfrac{G(P+E)\left(m + \dfrac{4\pi r^3}{c^2}P\right)}{c^2 r\left(r - \dfrac{2Gm}{c^2}\right)}, \\ \dfrac{dm}{dr} = 4\pi r^2 \dfrac{E}{c^2}. \end{cases}$$

$$\Delta P = \left(\frac{\partial P}{\partial r}\right)_1 \delta r = \frac{1}{\lambda}\left(\frac{\partial P}{\partial r}\right)_2 \delta r.$$

$$\Delta m = 4\pi r_*^2 \frac{E_1}{c^2}\delta r = 4\pi r_*^2 \frac{E_2}{c^2}\left[1-(\lambda-1)\frac{P_2}{E_2}\right]\frac{\delta r}{\lambda}.$$

Phase equilibrium

$$\begin{cases} P_{pt} = P_1(n_1) = P_2(n_2) \\ \mu_{pt} = \dfrac{P_1+E_1}{n_1} = \dfrac{P_2+E_2}{n_2} \end{cases}$$

$$E_2 = E_2(n_2, \zeta), \quad \zeta \equiv B \text{ for quark matter}$$

$$\Delta P = \frac{\delta\zeta}{\lambda-1}\left(\frac{\partial E_2}{\partial \zeta}\right), \quad \lambda \equiv \frac{n_2}{n_1}.$$

$$\begin{cases} \Delta P = \left(\dfrac{dP}{dr}\right)_2 \delta r + \left(\dfrac{dP}{dP_c}\right)_{r,\zeta} \delta P_c + \left(\dfrac{dP}{d\zeta}\right)_{r,P_c} \delta\zeta, \\ \Delta m = \left(\dfrac{dm}{dr}\right)_2 \delta r + \left(\dfrac{dm}{dP_c}\right)_{r,\zeta} \delta P_c + \left(\dfrac{dm}{d\zeta}\right)_{r,P_c} \delta\zeta. \end{cases}$$

The main equation

$$\frac{dP}{dr}\left[\frac{\lambda-1}{\lambda}\right]\delta r=\left(\frac{\partial E_2}{\partial \zeta}\right)\partial\zeta,$$

$$-\frac{dP}{dr}\left[\frac{\lambda-1}{\lambda}\right]\delta r=\left(\frac{\partial P}{\partial P_c}\right)_{r,\zeta}\delta P_c+\left(\frac{\partial P}{\partial \zeta}\right)_{r,P_c}\delta\zeta,$$

$$-\frac{dm}{dr}\left[\frac{P_2+E_2}{E_2}\right]\left[\frac{\lambda-1}{\lambda}\right]\delta r=\left(\frac{\partial m}{\partial P_c}\right)_{r,\zeta}\delta P_c+\left(\frac{\partial m}{\partial \zeta}\right)_{r,P_c}\delta\zeta.$$

Det = 0!

$$\left(\frac{\partial P}{\partial P_c}\right)_{r,\zeta}\left[\frac{dm}{dr}\frac{(P_2+E_2)}{E_2\frac{dP}{dr}}\left(\frac{\partial E_2}{\partial \zeta}\right)+\left(\frac{\partial m}{\partial \zeta}\right)_{r,P_c}\right]=\left(\frac{\partial m}{\partial P_c}\right)_{r,\zeta}\left[\left(\frac{\partial E_2}{\partial \zeta}\right)+\left(\frac{\partial P}{\partial \zeta}\right)_{r,P_c}\right].$$

Dimensionless form of the Main Equation

$$P = \alpha(E - E_0), \quad E_0 \equiv 4B, \quad \alpha_q = \frac{1}{3}$$

$$\rho = \frac{E}{E_0}, \quad x = \frac{r}{r_{\dim}}, \quad \mu = \frac{m}{m_{\dim}}$$

$$r_{\dim} = \frac{c^2}{\sqrt{4\pi G E_0}}, \quad m_{\dim} = \frac{c^4}{G\sqrt{4\pi G E_0}}.$$

Equilibrium equations:

$$\begin{cases} \alpha \dfrac{d\rho}{dx} = -\left[\rho + \alpha(\rho - 1)\right] \dfrac{\mu + \alpha x^3(\rho - 1)}{x(x - 2\mu)}, \\ \dfrac{d\mu}{dx} = x^2 \rho. \end{cases}$$

$$\left(\frac{\partial \mu}{\partial \rho_c}\right)_x \left[\rho - \frac{\alpha}{1+\alpha} + \frac{x}{2}\frac{d\rho}{dx}\right]\frac{d\rho}{dx} = x^2 \left(\frac{\partial \rho}{\partial \rho_c}\right)_x \left[\rho - \frac{\alpha}{1+\alpha} + \frac{x}{2}\frac{d\rho}{dx}\left(\rho - \frac{\mu}{x^3}\right)\right]$$

Homology-invariant variables

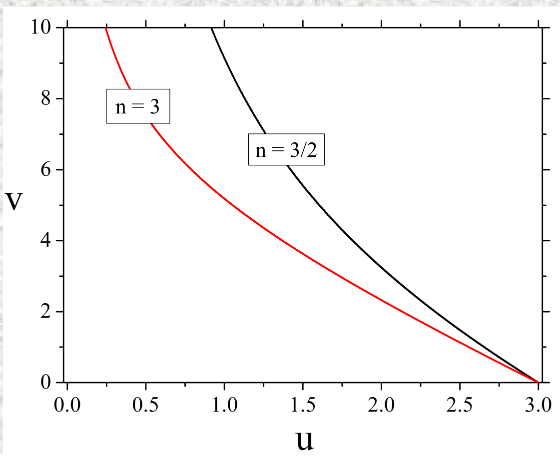
For some specific EOSes TWO TOV-equations of equilibrium can be combined to ONE differential equation with homological variables.

$$\begin{cases} x\dfrac{du}{dx} = f_u(u,v), \\ x\dfrac{dv}{dx} = f_v(u,v). \end{cases} \Rightarrow \quad \frac{dv}{du} = \frac{f_v(u,v)}{f_u(u,v)} = f(u,v).$$

Examples:

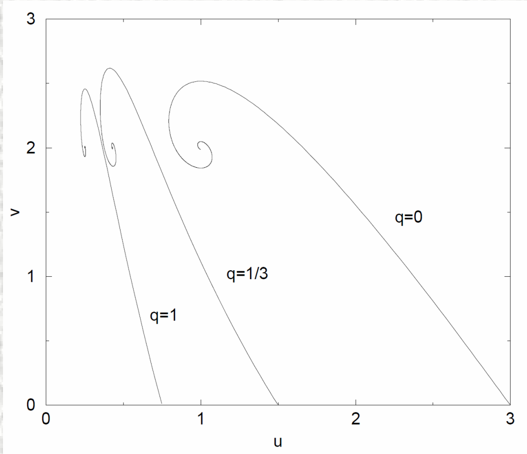
Newtonian limit, polytropes:

$$\begin{cases} u = \dfrac{d\ln m}{d\ln r} = \dfrac{4\pi r^3 \rho}{m}, \\ v = -\dfrac{d\ln P}{d\ln r} = \dfrac{Gm\rho}{rP}. \end{cases} \Rightarrow \quad \frac{u}{v}\frac{dv}{du} = -\frac{u + \dfrac{v}{1+n} - 1}{u + \dfrac{nv}{1+n} - 3}$$



GR, P=qE

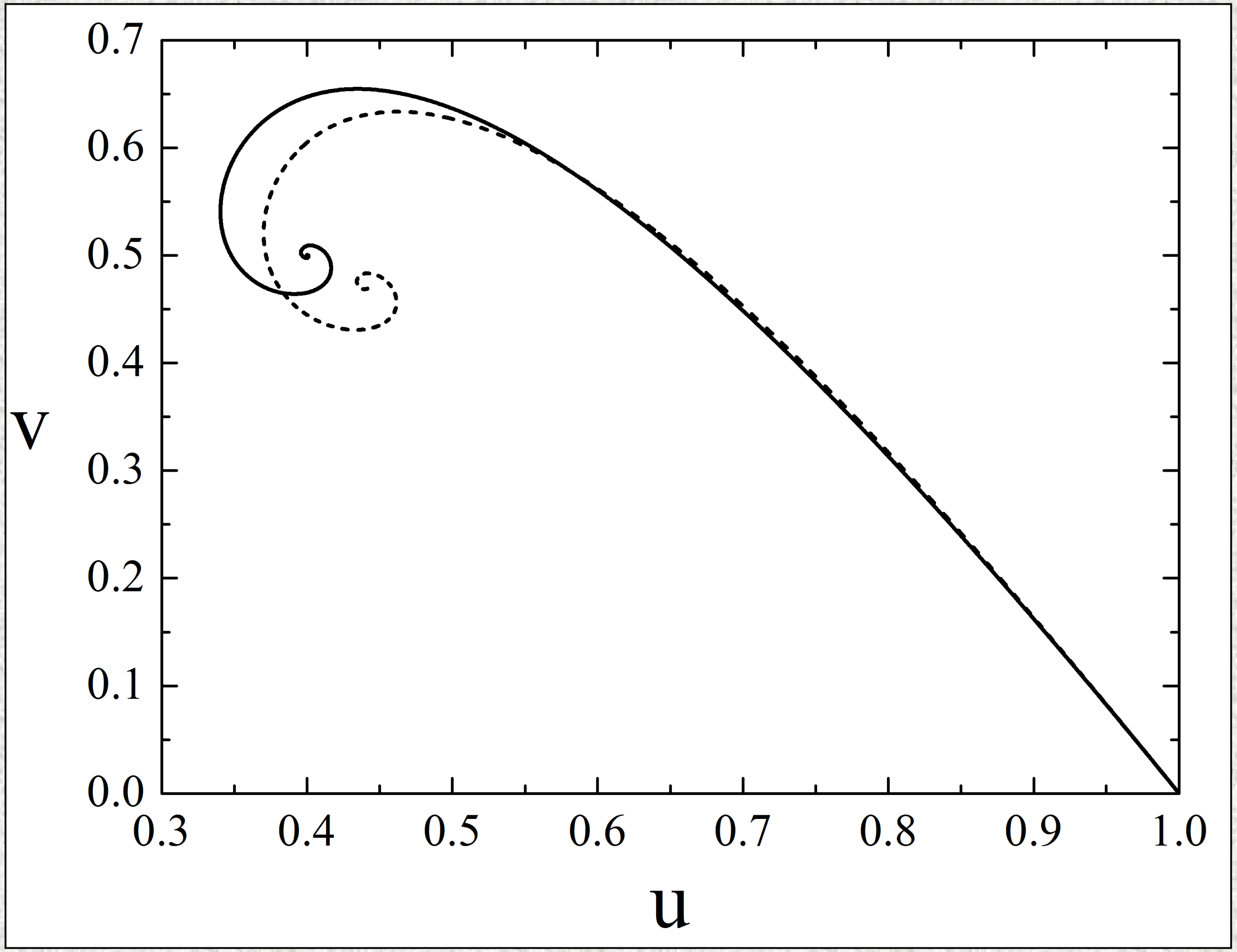
$$\frac{u}{v}\frac{dv}{du} = \frac{-1 - \frac{2q}{1+q}v + (1+3q)u + \frac{q(3+5q)}{1+q}uv + \frac{2q^2(1-q)}{(1+q)^2}uv^2}{3 - \frac{1-q}{1+q}v - (1+3q)u - \frac{q(3+q)}{1+q}uv - \frac{4q^2}{(1+q)^2}uv^2}.$$



From P.-H. Chavanis, Astron. Astrophys.,381, (2002)

“Almost” homological variables

$$\begin{cases} v = \dfrac{\mu + \alpha x^3(\rho - 1)}{x - 2\mu}, \\ u = \dfrac{x^3\left[\rho + \alpha(\rho - 1)\right]}{3\mu + \alpha x^3(\rho - 1)}. \end{cases}$$



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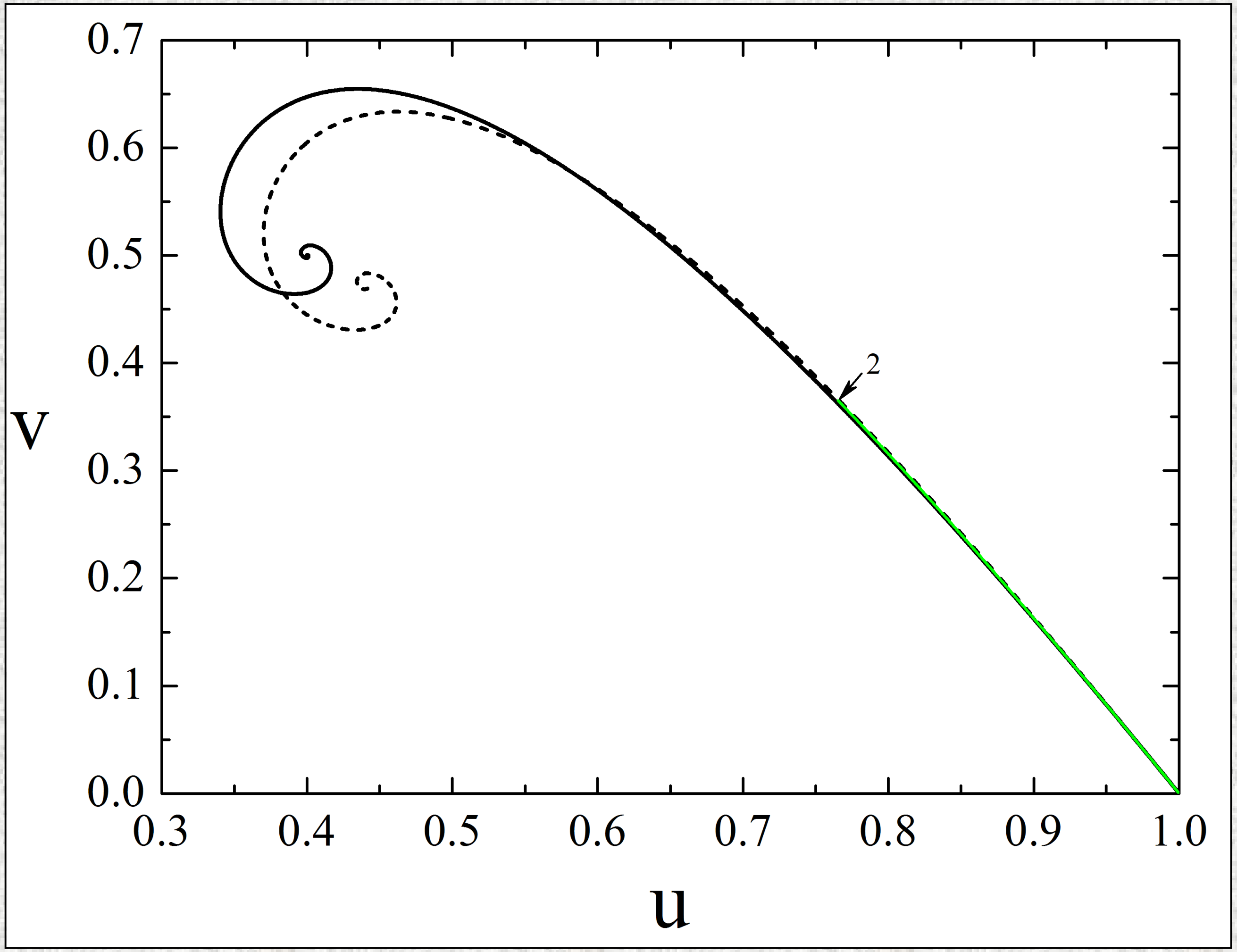

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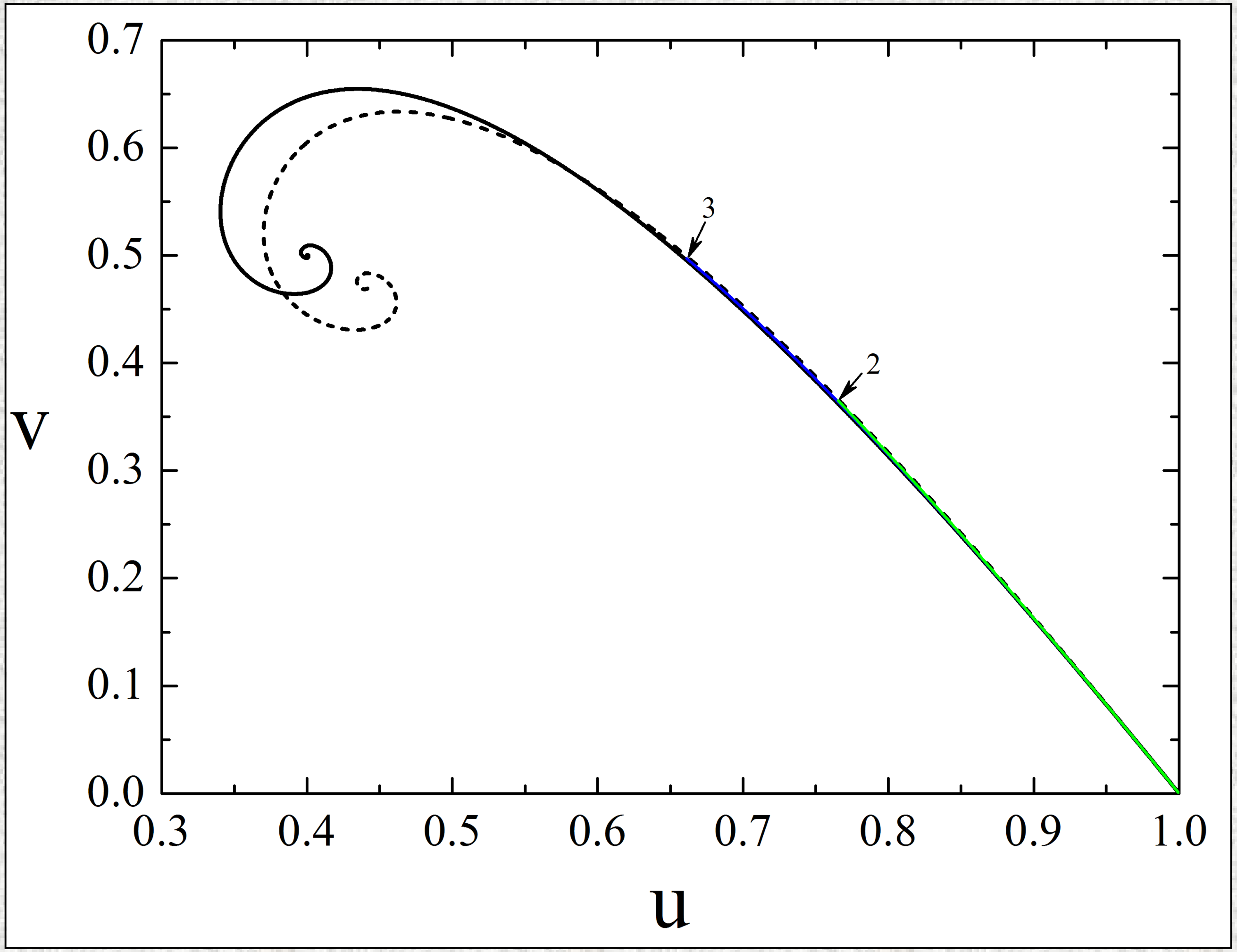

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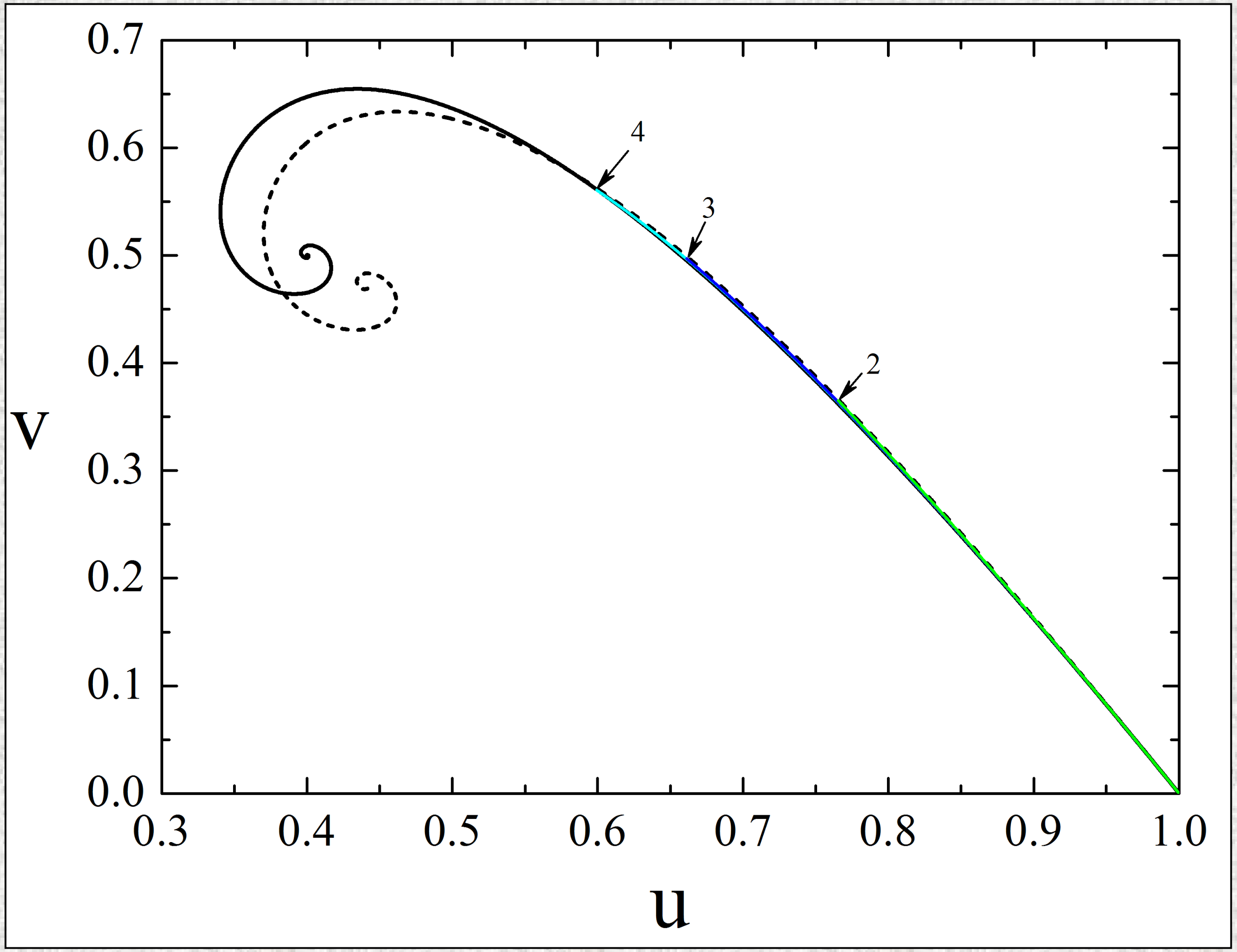

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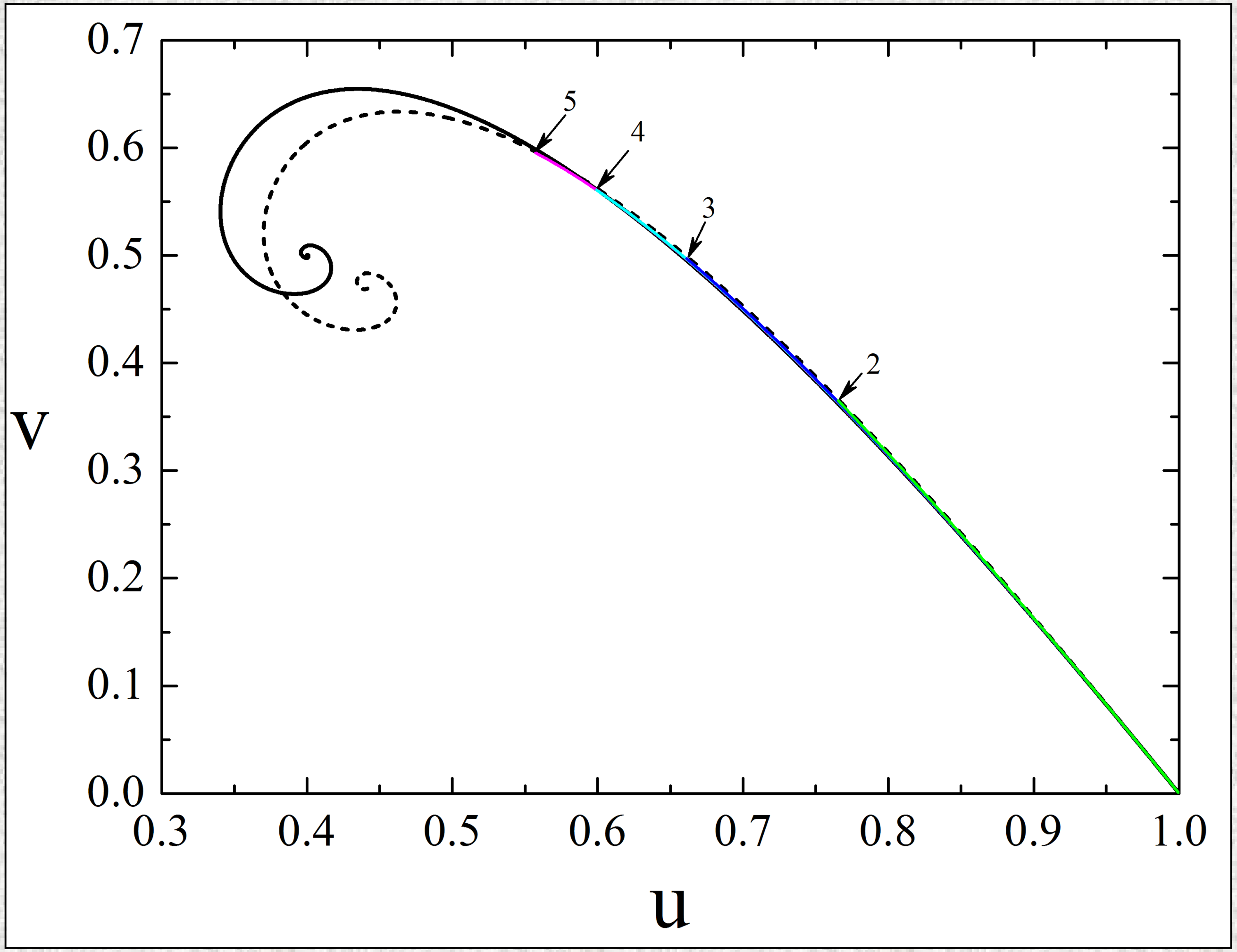

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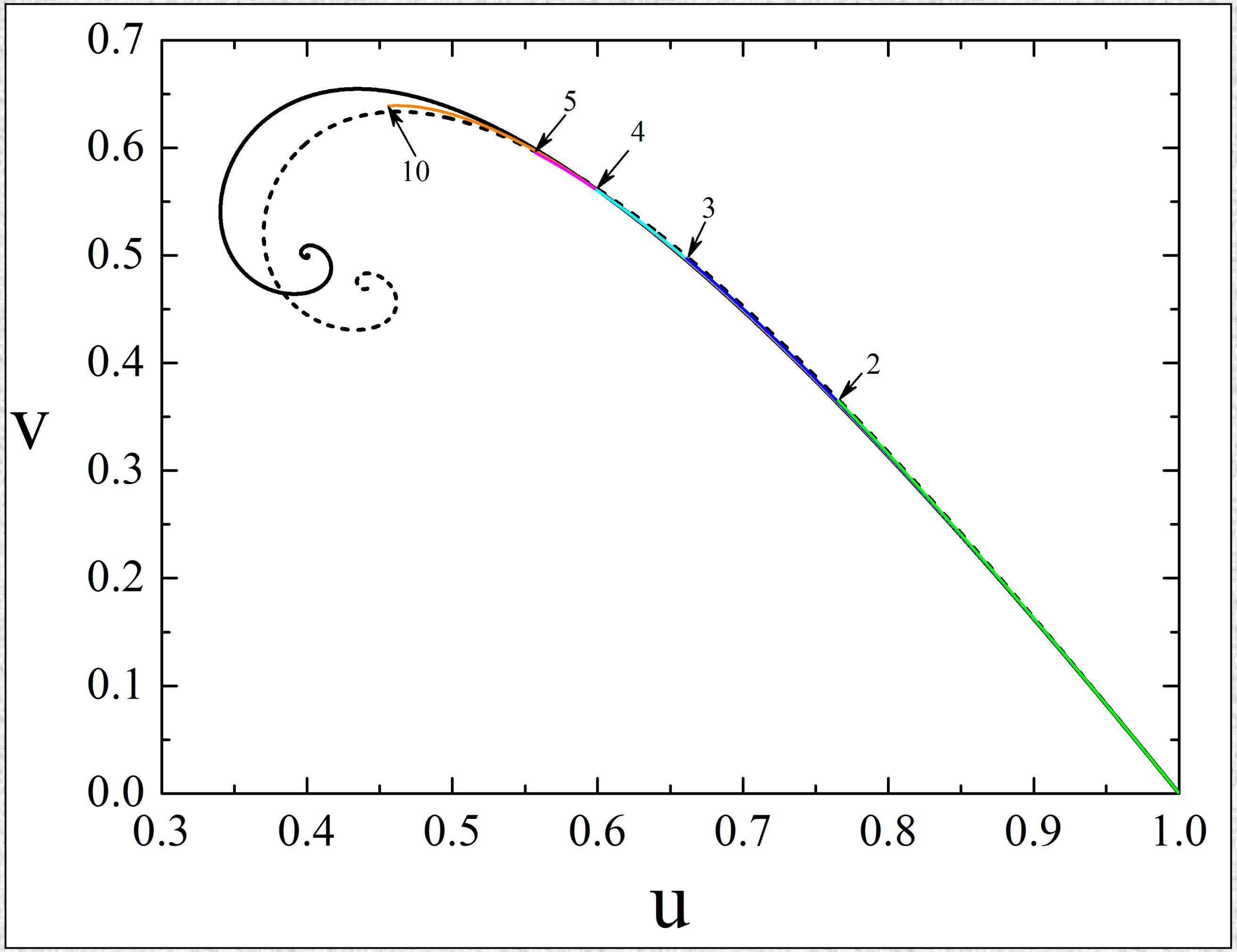
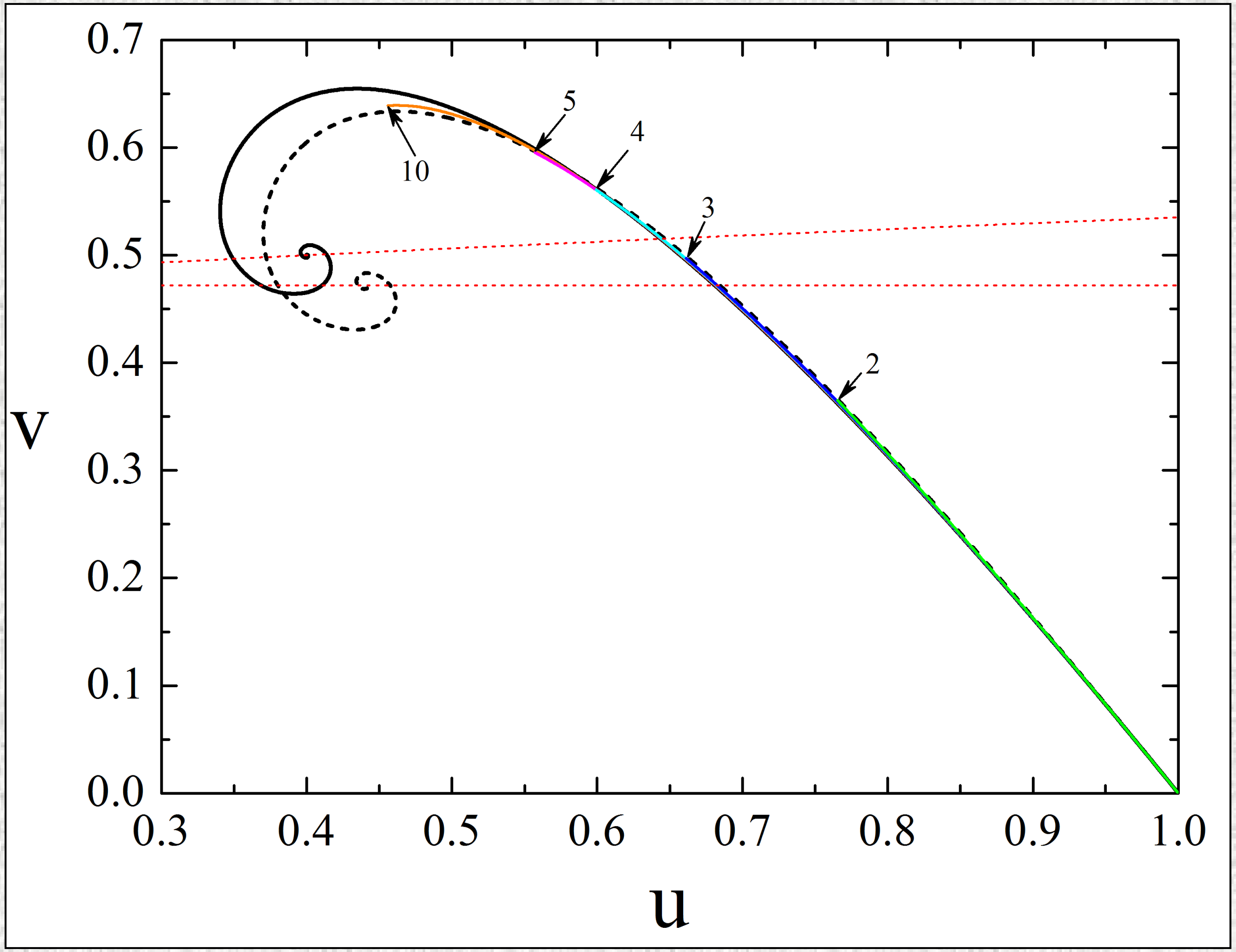
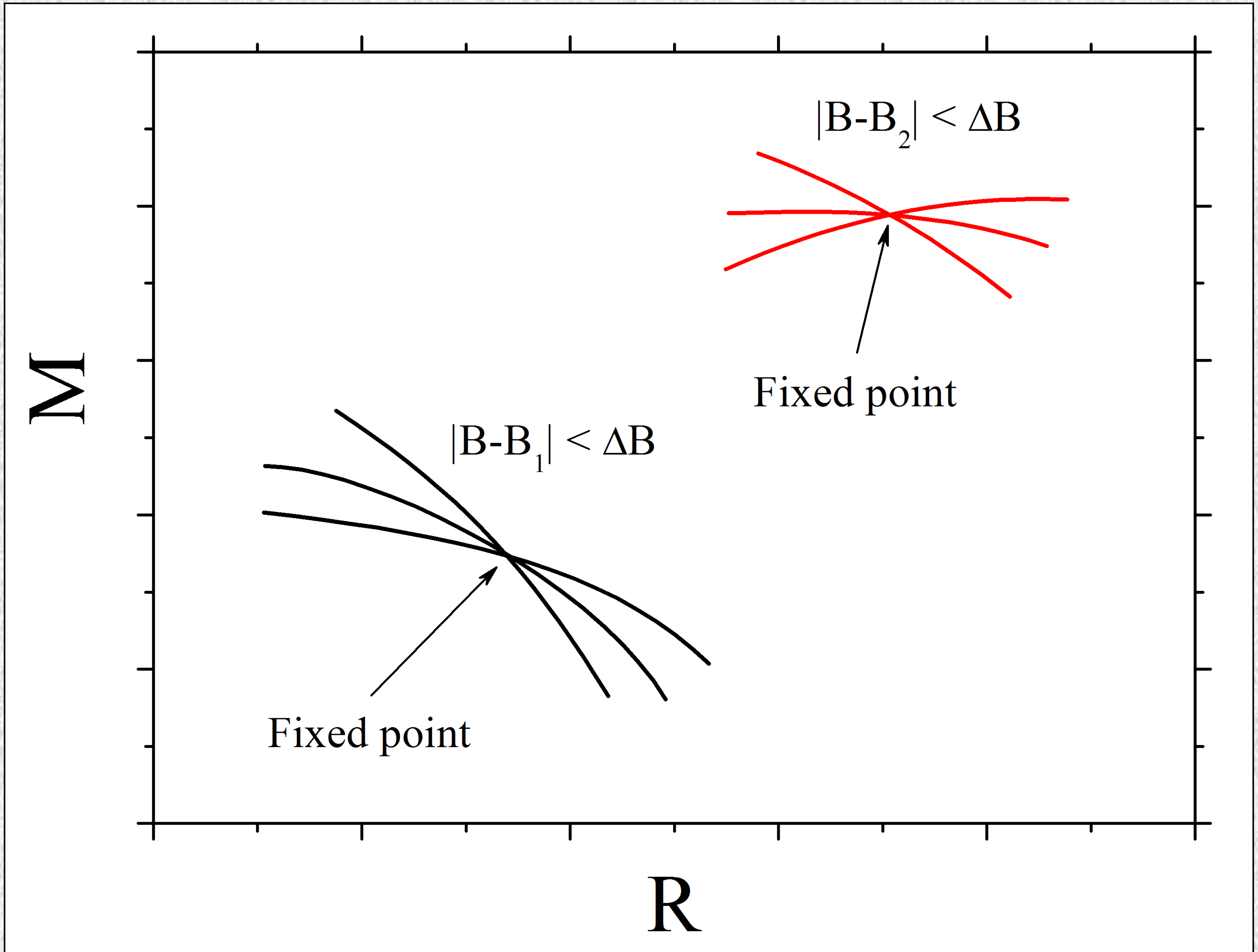


$$\left(\frac{\partial\mu}{\partial\rho_c}\right)_x\left[\rho-\frac{\alpha}{1+\alpha}+\frac{x}{2}\frac{d\rho}{dx}\right]\frac{d\rho}{dx}=x^2\left(\frac{\partial\rho}{\partial\rho_c}\right)_x\left[\rho-\frac{\alpha}{1+\alpha}+\frac{x}{2}\frac{d\rho}{dx}\left(\rho-\frac{\mu}{x^3}\right)\right]$$



$$u_*=\left[v_*^2(3+\alpha)+v_*(3-\alpha)-6\alpha\right]\frac{1+(1+\alpha)(\rho_*-1)}{4\alpha^2(\rho_*-1)(1-v_*)(3-v_*)}$$

Global structure of fixed points?



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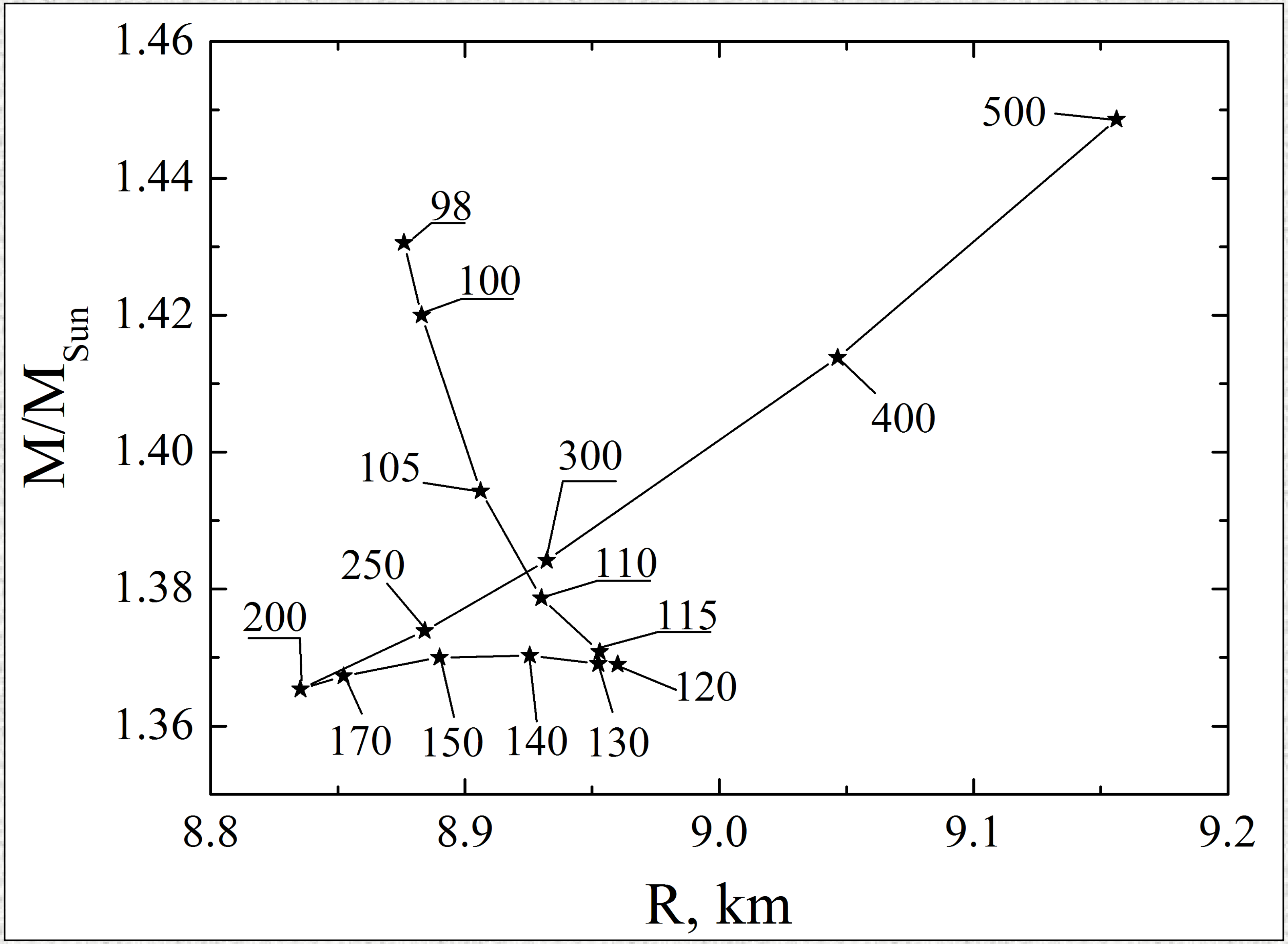
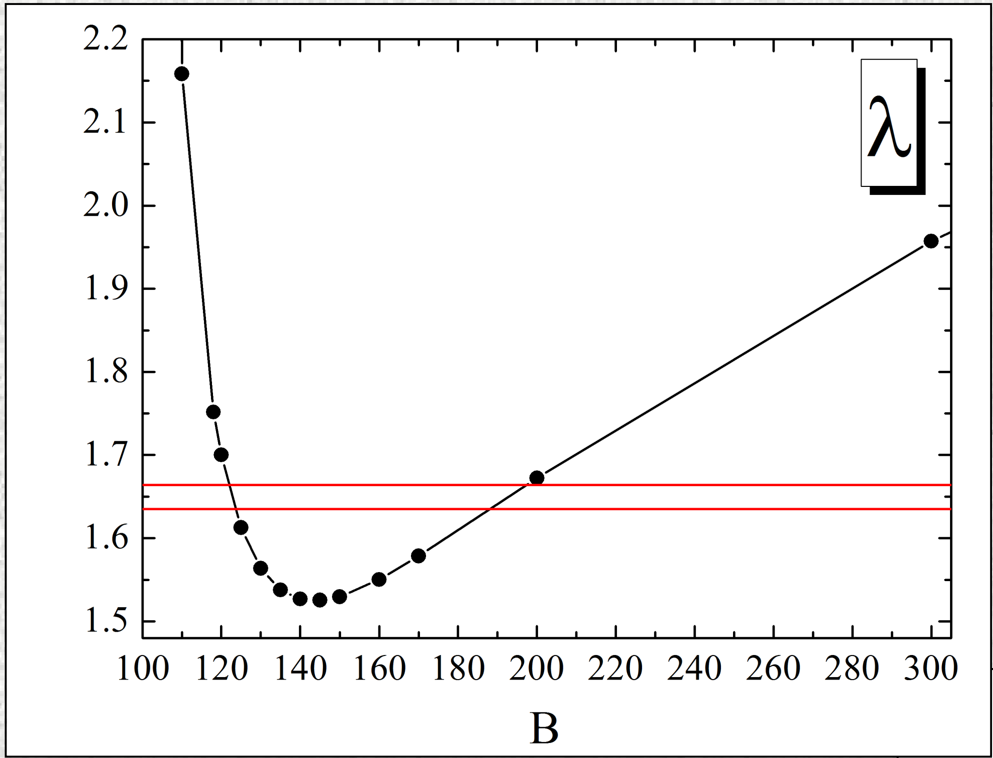
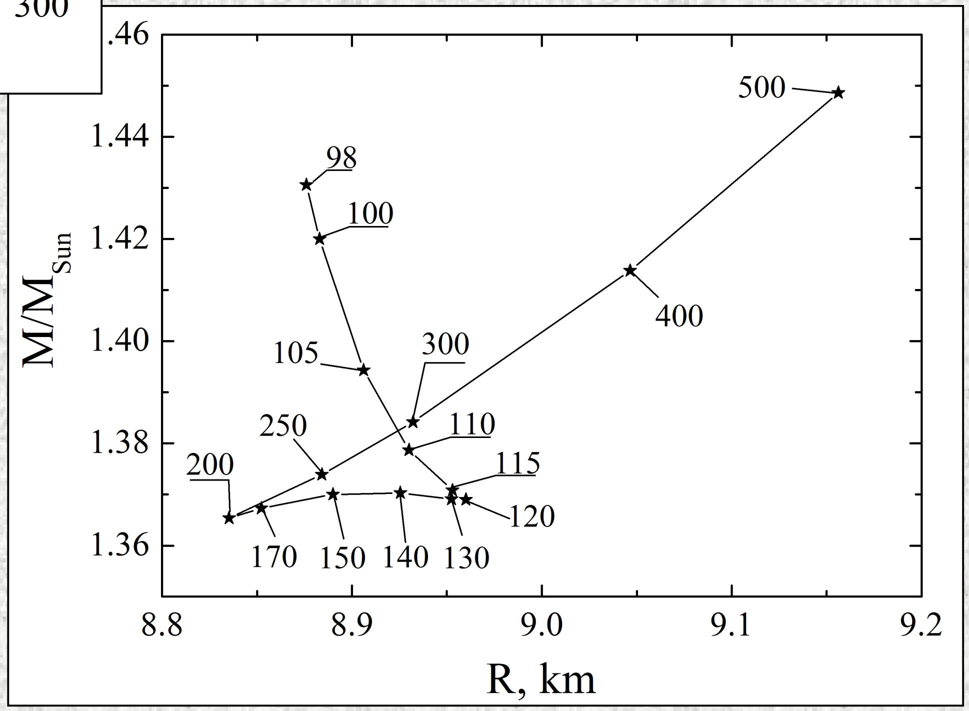




The condition that a small shift from a fixed point bring us to another fixed point

$$\lambda = \frac{u(3-v)\left[(1+v)^2(3-v)+8(1-v^2)\alpha-(3-v)^2\alpha^2\right]}{(1+v)(7v^2-6v+3)+8(1-v^2)(3-v)\alpha-(3-v)^3\alpha^2}$$

$$\lambda \equiv \frac{n_2}{n_1}$$



Open questions:

Other (non-linear) EOS as the solution of the main equation?
Other topology of fixed-points because different envelope?