Random walks of photons in relativistic flow and its application to gamma-ray burst

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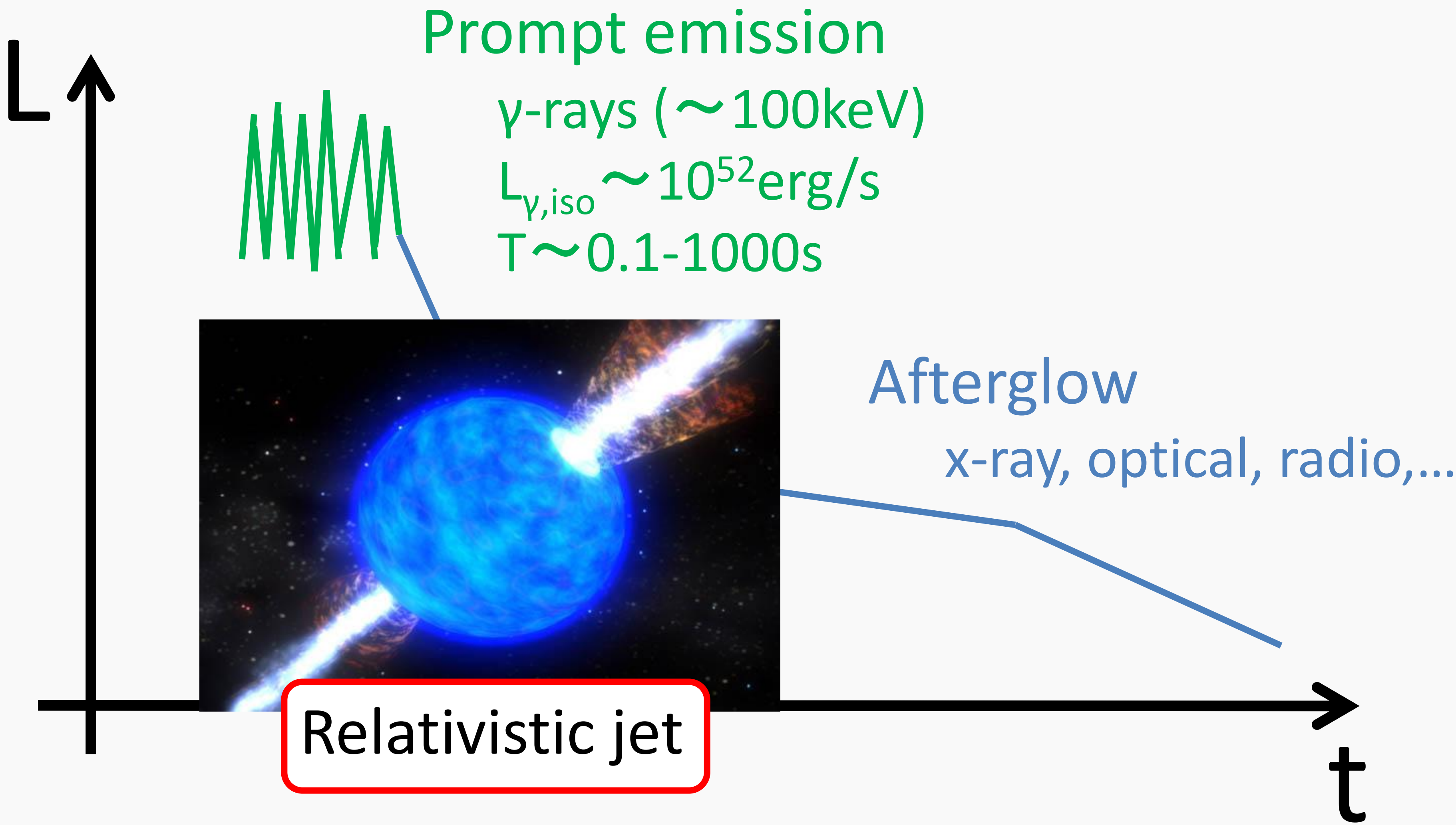
Outline

- Introduction
- Random walks in relativistic flow
- Application to gamma-ray burst
- Summary

Introduction

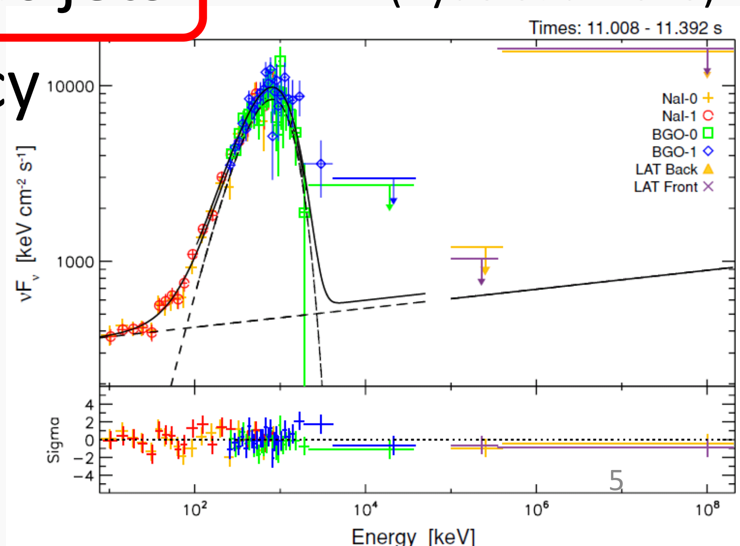
Gamma-Ray Burst (GRB)



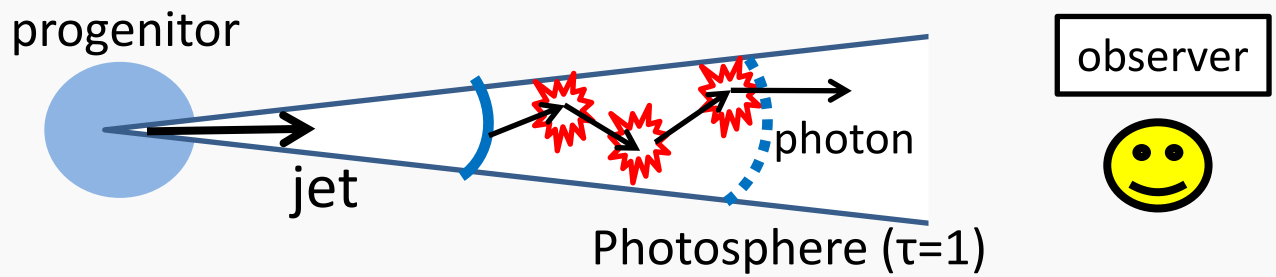
Models for the prompt emission

- Internal shock model
  - A standard scenario for a long time.
  - Some problems about the radiative efficiency and the low energy photon index
- Photospheric (thermal emission) model
  - Thermal emission from relativistic jets
  - (possibly) high radiative efficiency
  - Some GRBs exhibit blackbody like feature (e.g., GRB090902B).

(Ryde et al 2010)



Thermal emission from GRB jet



- Photons are not produced at the photosphere
- We have to calculate radiative transfer
- We need to know where the photons are produced
- We construct the expression for effective optical depth in relativistic flow considering random walk process in relativistic flow

Random walks in relativistic flow

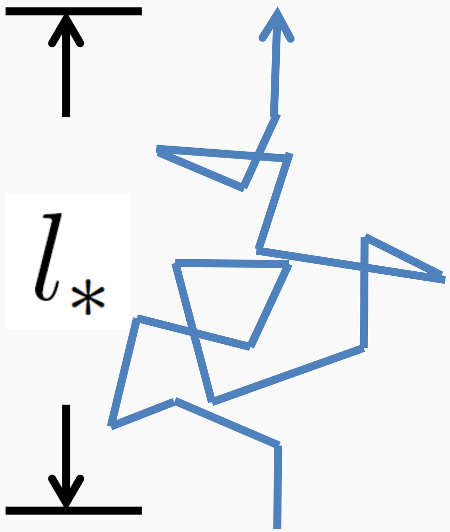
Random walks of photons

- Displacement of a photon

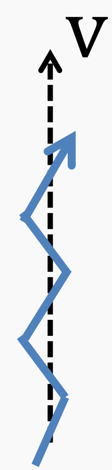
$$\mathbf{R} = \mathbf{r}_1 + \mathbf{r}_2 + \cdots + \mathbf{r}_N$$

- The average net displacement

$$l_*^2 \equiv \langle \mathbf{R}^2 \rangle = \sum_{i=1}^{N} \langle \mathbf{r}_i^2 \rangle + \sum_{\substack{i,j \\ i \neq j}}^{N} \langle \mathbf{r}_i \cdot \mathbf{r}_j \rangle .$$



- The second term is 0 in the static medium
- But it is not 0 in the relativistic flow
  (due to the relativistic beaming effect)



Random walks of photons

- Taking into account relativistic effect

$$l_*^2 = N\frac{2}{3}\Gamma^2(\beta^2+3)l_0^2 + N(N-1)(\Gamma\beta)^2 l_0^2$$

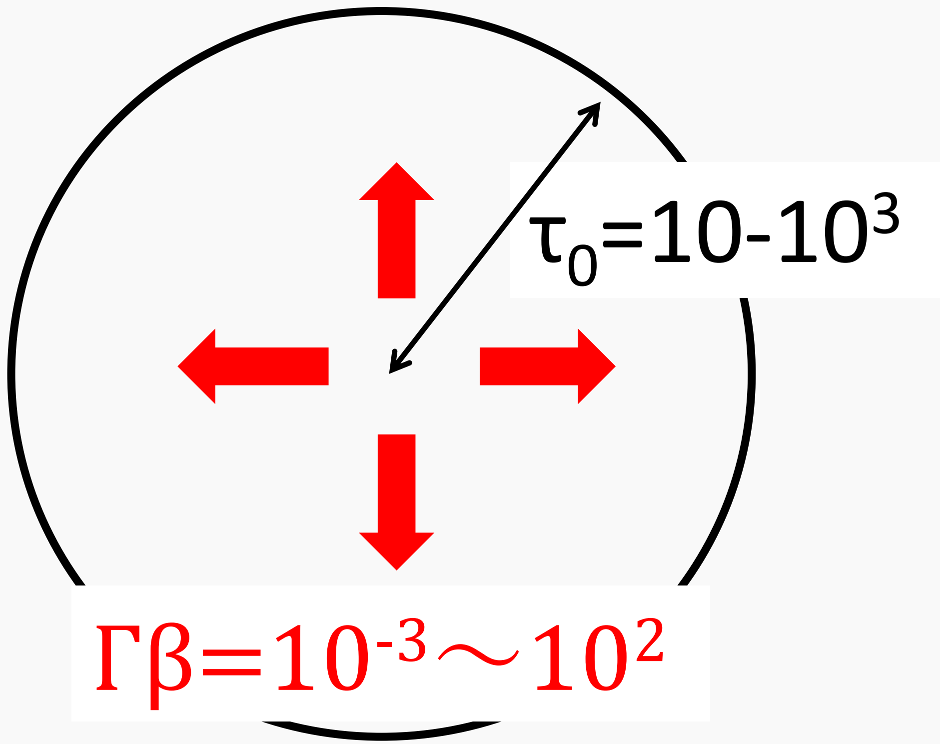
- If we set $l_* = L$ and introduce $\tau_0 \equiv L/l_0$

$$N = \frac{1}{2a}(\sqrt{b^2+4a\tau_0^2} - b)$$

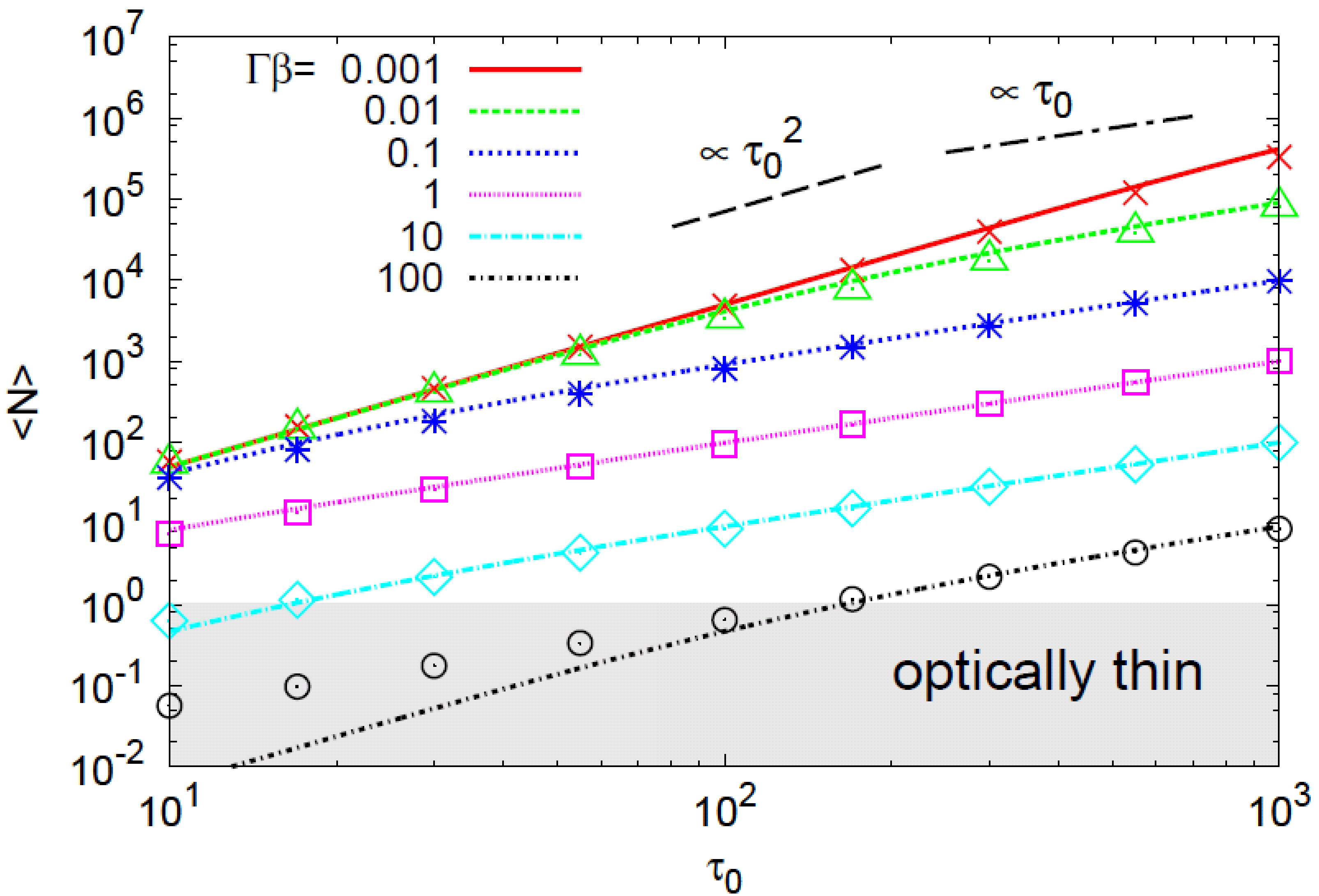
where $a = (\Gamma\beta)^2$ and $b = \Gamma^2(2-\beta^2/3)$.

Comparison with numerical simulation

- Monte-Carlo simulation of photon propagation
- Calculate number of scatterings



Comparison with numerical simulation



The effective optical depth

- The effective optical depth τ_*

  For the static medium (Rybicki & Lightman 79)

$$\tau_*^{\rm NR} \sim \sqrt{\tau_{\rm a}(\tau_{\rm a}+\tau_{\rm s})}$$

  For the relativistic medium

$$\tau_*^{\rm R} = \left\{ \frac{\Gamma^2}{3}(\beta^2+3) + (\Gamma\beta)^2 \frac{\tau_{\rm s}}{\tau_{\rm a}} \right\}^{-1/2} \frac{\sqrt{\tau_{\rm a}(\tau_{\rm a}+\tau_{\rm s})}}{\Gamma(1-\beta\cos\theta_{\rm v})}$$

$$\tau_{\rm a} = \Gamma(1-\beta\cos\theta_{\rm v})\alpha' L \ , \quad \tau_{\rm s} = \Gamma(1-\beta\cos\theta_{\rm v})\sigma' L$$

  In the non-relativistic limit, $\tau_*^{\rm R} \to \tau_*^{\rm NR}$

  In the relativistic limit, $\tau_*^{\rm R} \to 2\,\tau_{\rm a}$ for $\Theta=0$

Application to Gamma-Ray Burst

Calculation method

Hydrodynamical simulation

↓

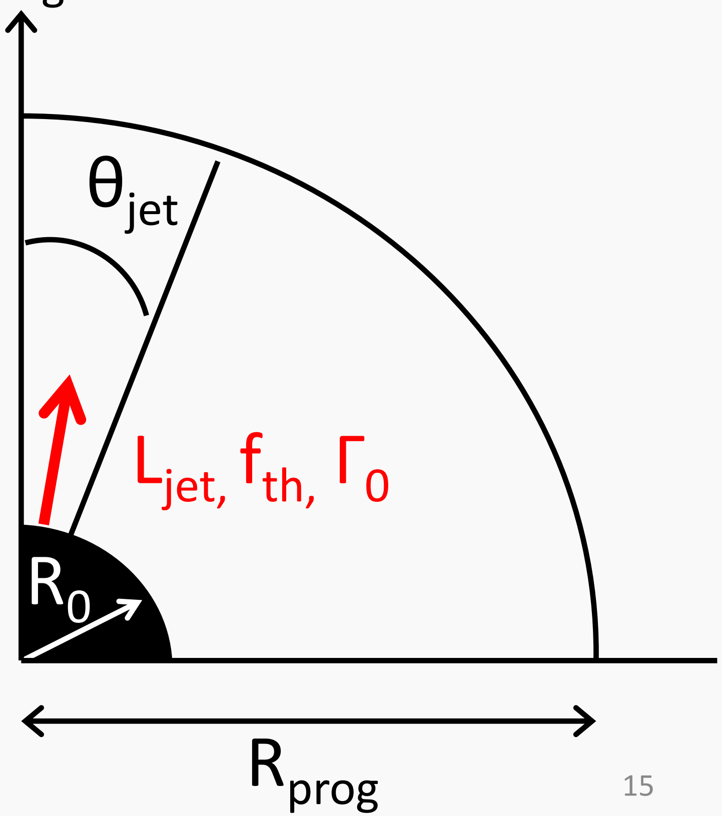
Estimation of the photon production site

↓

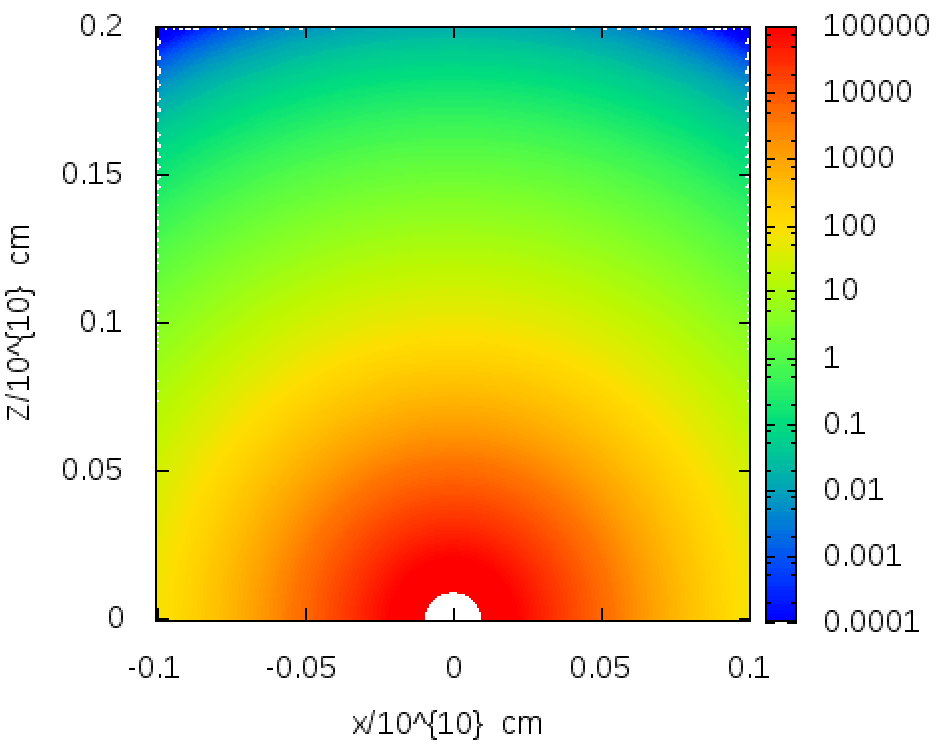
Radiative transfer simulation

Hydrodynamical simulation

- ✓ 2D relativistic hydrodynamics (Tominaga 2009)
- ✓ Setup
  - Progenitor: $15M_{sun}$ WR star ($R_{prog} \sim 2.3 \times 10^{10}$cm)
  - $\Gamma_0=5$
  - $\Theta_{jet}=10^\circ$
  - $L_{jet}=5.3 \times 10^{50}$ erg s^{-1}
  - $f_{th}=0.9925$ ($e_{int}/\rho c^2=80$)
  - $(\log r, \theta) = (600, 150)$ grids from $R_0=10^9$cm



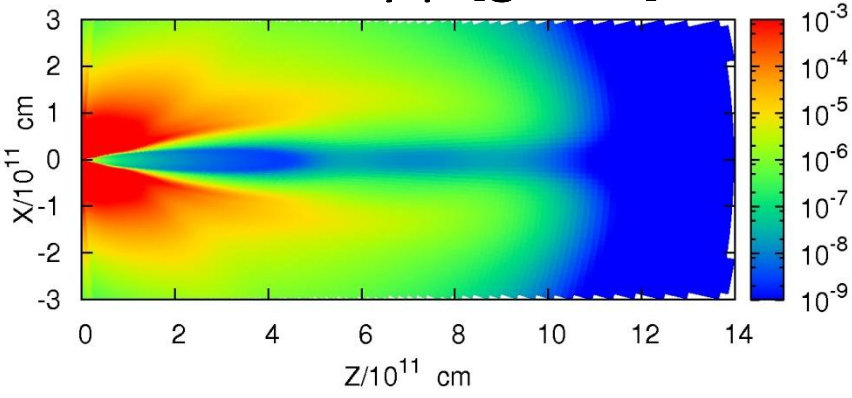
Hydrodynamical simulation



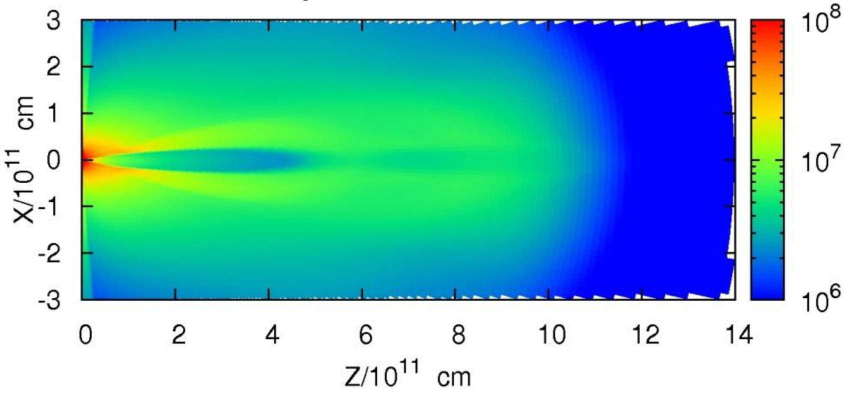
Hydrodynamical simulation

- We use a snapshot at 40s for the structures of the jet and cocoon.

Density ρ [g/cm^3]



Temperature T [K]



The photon production site

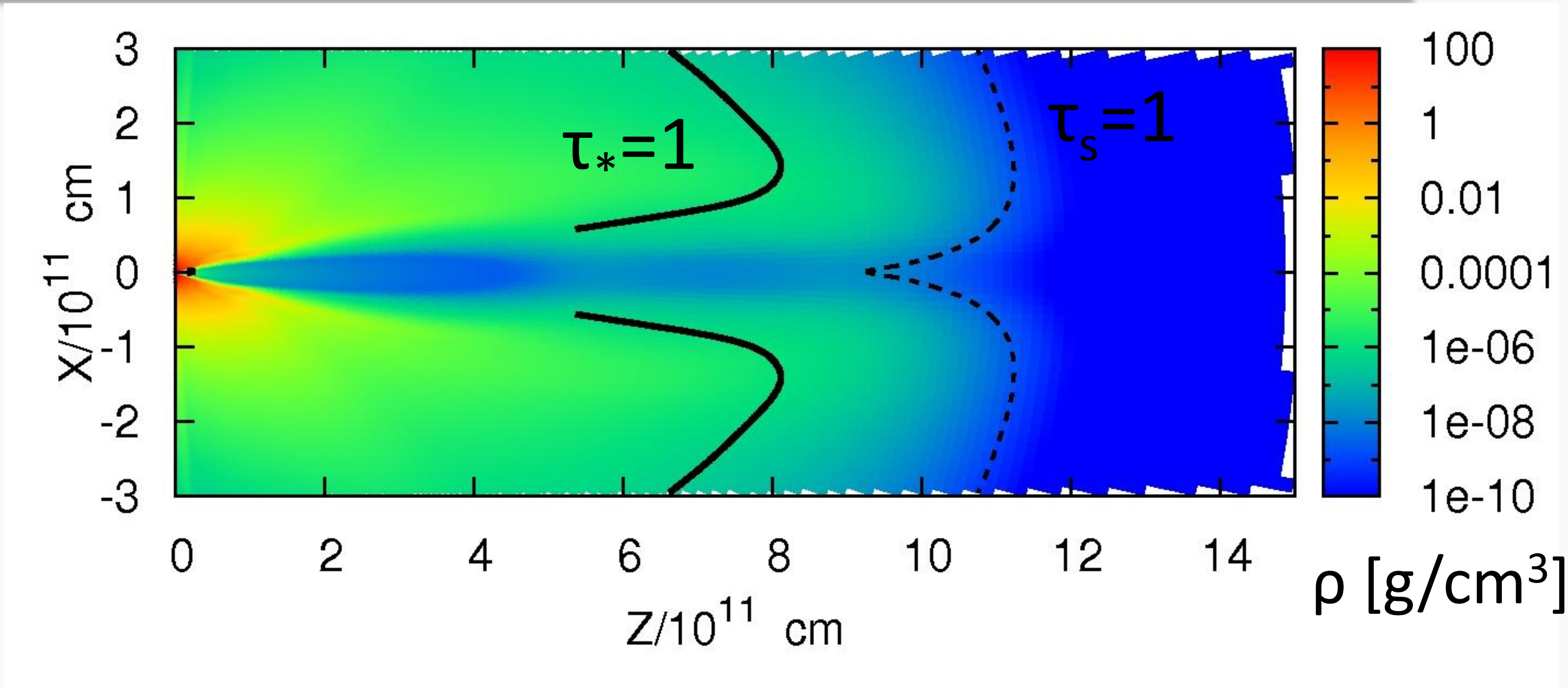
- τ_* to a radius R_*

$$\tau_* = \int_{R_*}^{\infty} \left\{ \frac{\Gamma^2}{3}(\beta^2+3) + (\Gamma\beta)^2 \frac{\sigma'}{\alpha'} \right\}^{-1/2} \sqrt{\alpha'(\alpha'+\sigma')}dr$$

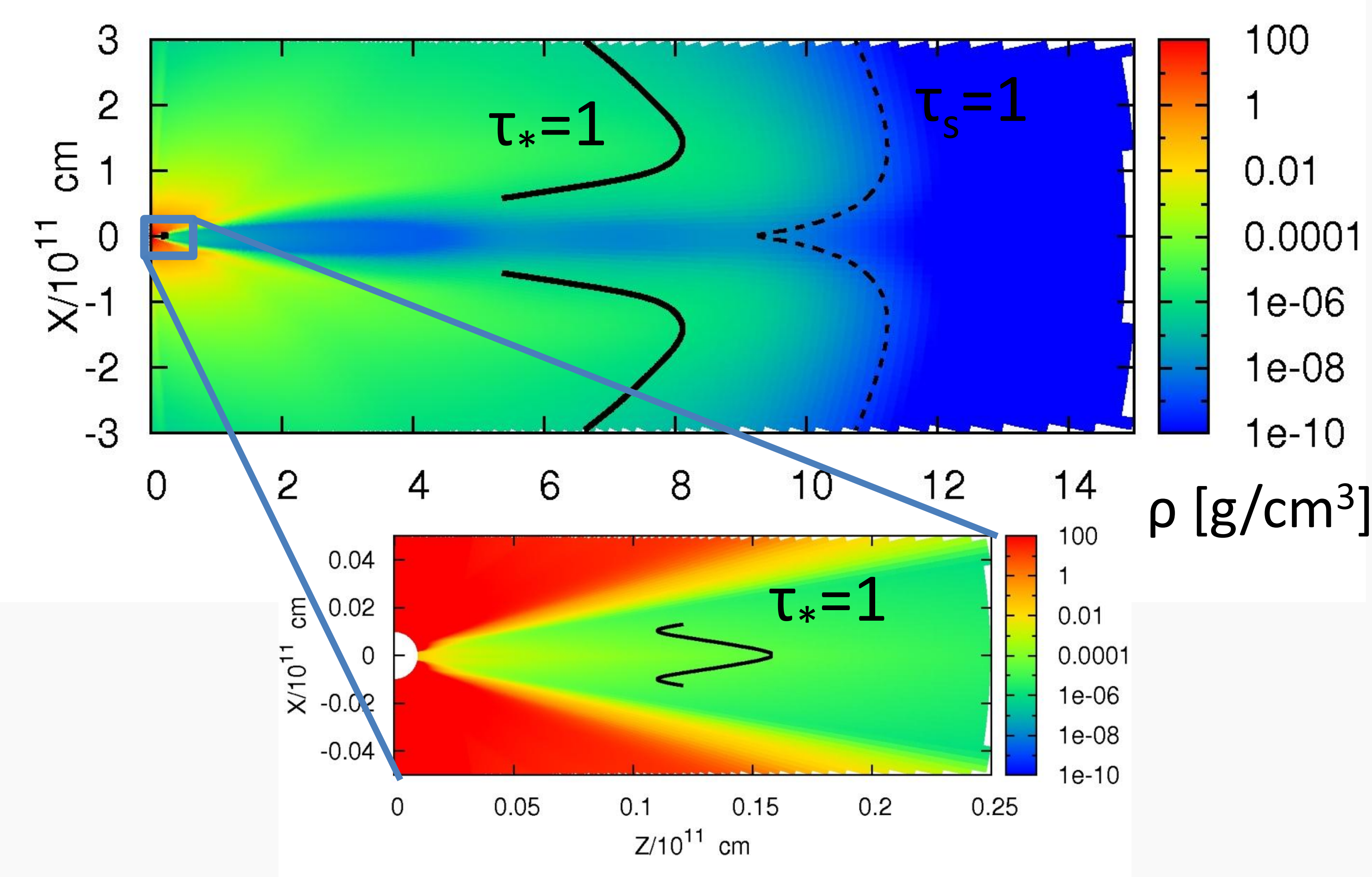
- σ’ : electron scattering
- α’ includes
  - Free-free absorption ($e + p + \gamma \rightarrow e + p$)
  - Double Compton absorption ($\gamma + \gamma + e \rightarrow \gamma + e$)

We find the R_* which satisfies $\tau_* = 1$

The photon production site



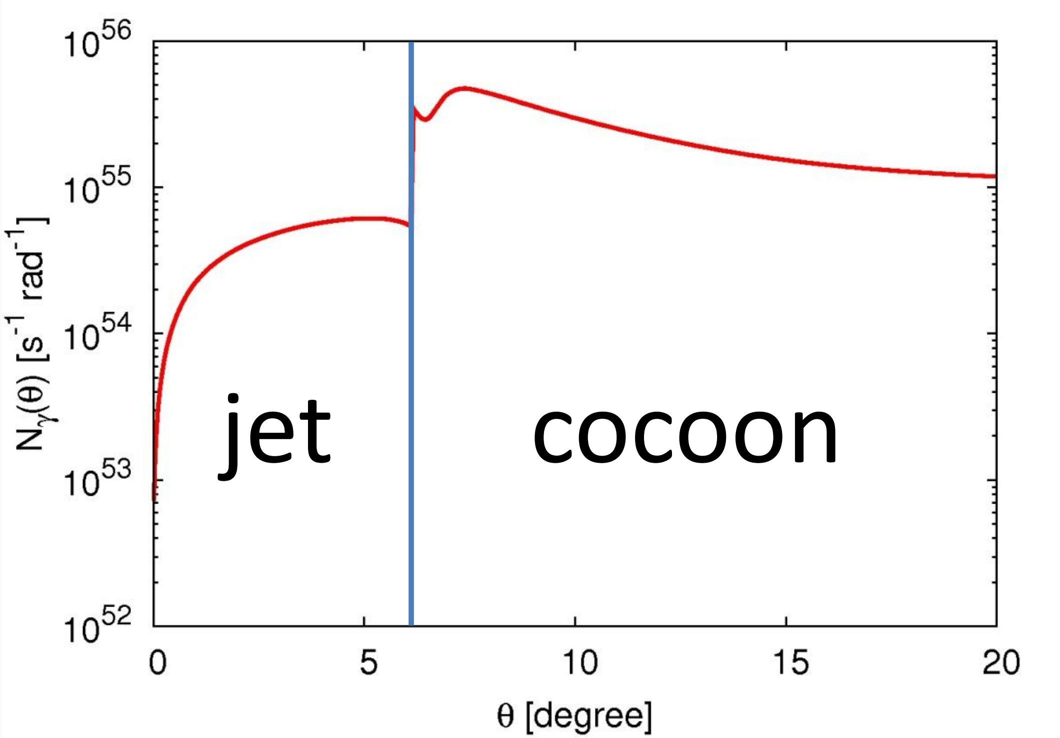
The photon production site



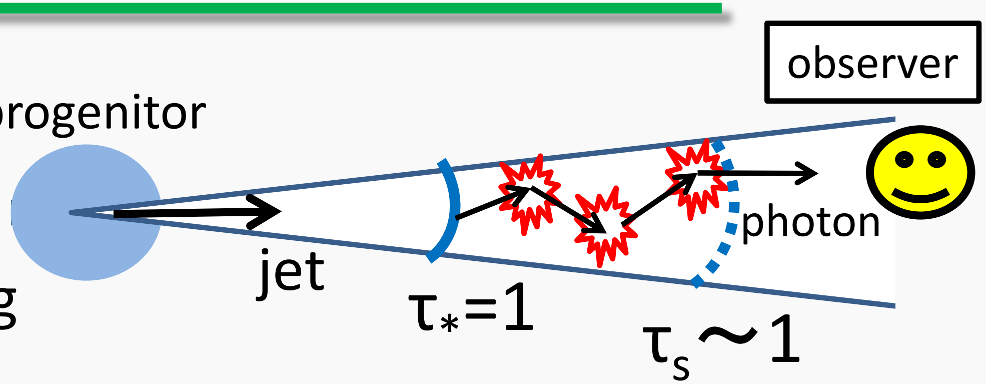
The photon production site

- The number of emitted photons:

$$N_\gamma(\theta) = 16\pi^2\Gamma(3)\zeta(3)\left(\frac{kT_*}{hc}\right)^3 R_*^2 \sin\theta_*$$



Radiative transfer



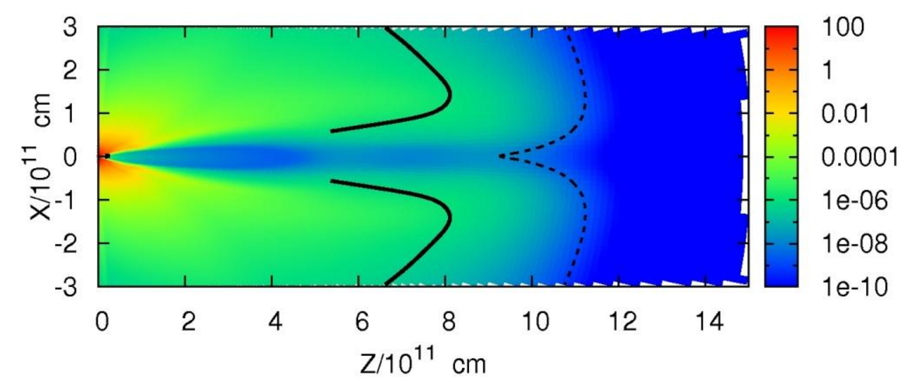
✓ Numerical code

- Monte Carlo method
- Calculate Compton scattering
- Photons are injected at $\tau_* = 1$

✓ Photon injection

- Spatial distribution: $N_\gamma(\Theta)$
- Planck distribution with local plasma temperatures
- Isotropic in the comoving frame

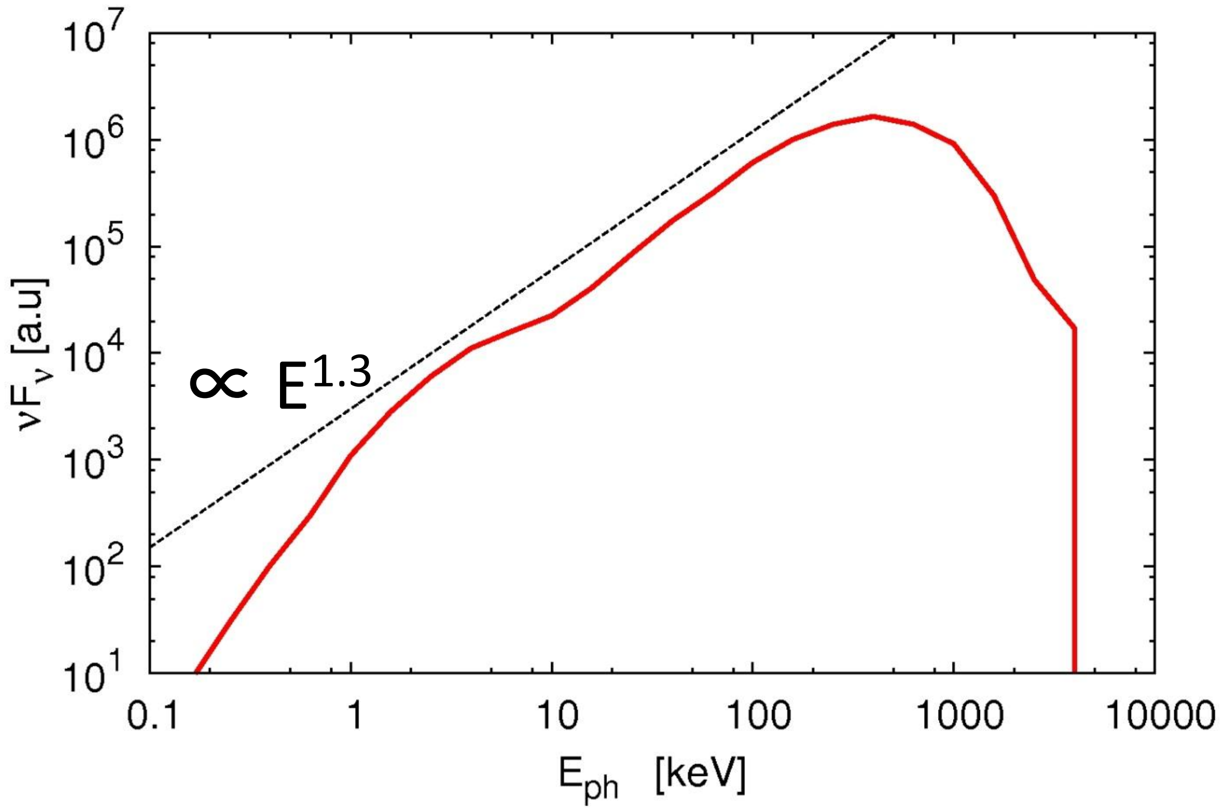
We use a snapshot at t=40s for the jet and cocoon structure.



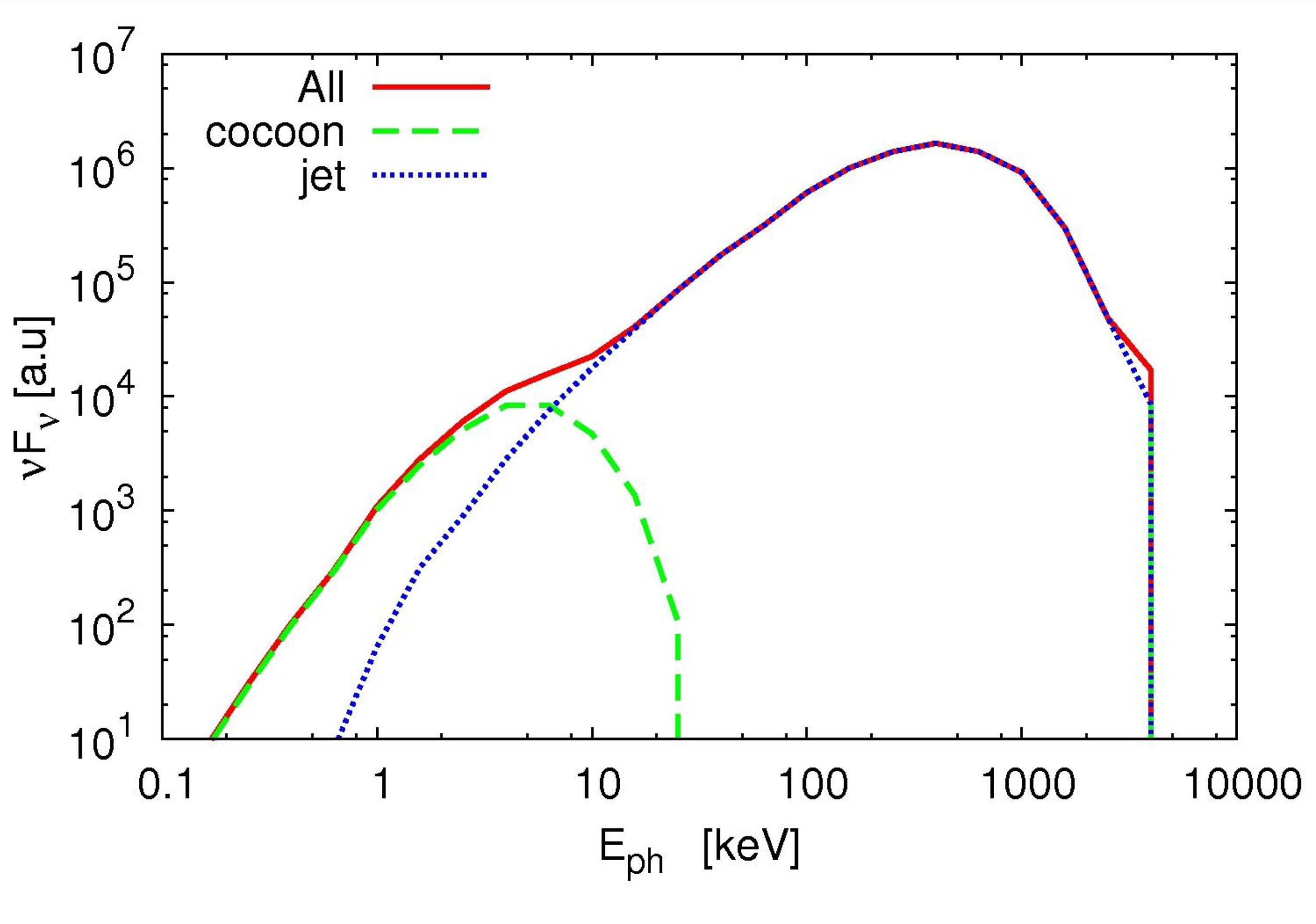
Results

Observed spectrum

- $E_{peak} \sim 450$keV
- A bump like feature at low energies
- At the low energy, $\nu F_\nu \propto E^{1.3}$ $\rightarrow N_\nu \propto E^{-0.7}$
- No high energy PL

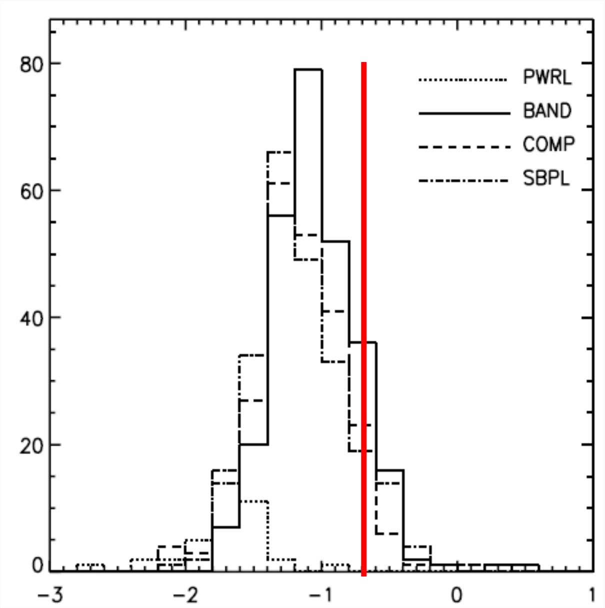


Origin of the bump?

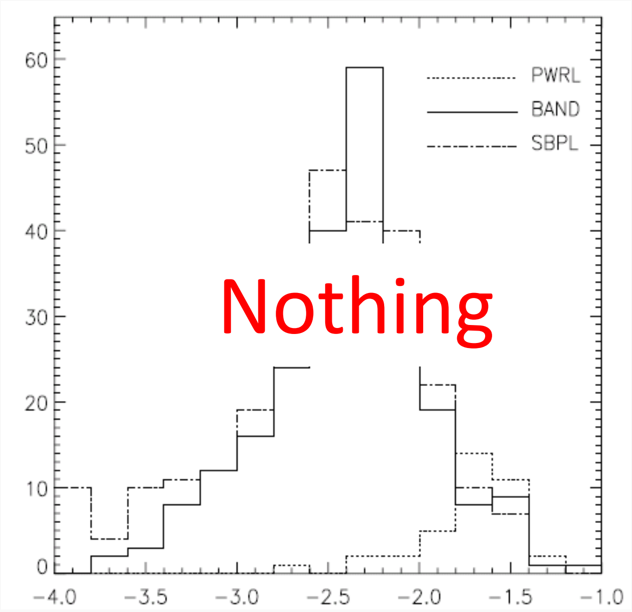


Comparison with the observations

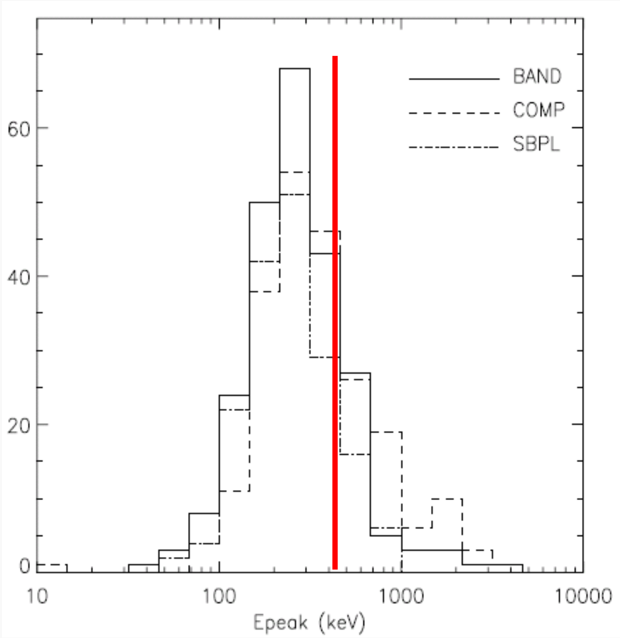
Kaneko et al 2006



low energy index (α)



high energy index (β)



peak energy (E_{peak})

Summary

Summary

- ✓ We constructed the expression for effective optical depth in relativistic flow.
- ✓ We calculated radiative transfer for the thermal radiaiton from GRB jet.
- ✓ Both the jet and cocoon components constitute the observed spectrum.
- ✓ The low energy index may be determined by the relative brightness of these two components.